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A SURVEY OF PROGRESS IN GRAPH THEORY IN THE SOVIET UNION

By: JAMES TURNER WILLIAM H. KAUTZ

Prepared for:

DIRECTORATE OF INFORMATION SCIENCES
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
OFFICE OF AEROSPACE RESEARCH
ARLINGTON, VIRGINIA 22209

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Approved: D. R. BROWN, DIRECTOR
Information Science Laboratory

TORBEN MEISLING, EXECUTIVE DIRECTOR
Information Science and Engineering Division

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ABSTRACT

This report describes the results of a comprehensive technical survey of all published Soviet literature in graph theory and its applications--more than 230 technical articles appearing up to June 1968. The purpose of this report is to draw attention to this collection of results, which are not well known in the West, and to summarize the significant contributions. Particular emphasis is placed upon those results that fill gaps or augment the body of knowledge about graph theory as familiar to non-Soviet specialists.

Although Soviet activity in graph theory and its applications lags behind the corresponding Western work in both quality and quantity, the level of activity is increasing rapidly and there are many excellent Soviet contributions to the theory. The best Soviet work has been concerned with bounds on numerical indices associated with graphs, properties of algebraic structures associated with graphs, and operations on graphs. Very little Soviet work has been reported on connectivity properties of graphs, matroid theory, the exact enumeration of graphs having prescribed properties, isomorphism testing, graph coloring, and the use of graphs for modeling in social sciences. A complete bibliography is given at the end of the report.

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CONTENTS (Concluded)

I INTRODUCTION

A. Need for the Survey

The field of graph theory and its applications is a rapidly expanding area of mathematical activity. Some measure of this expansion is indicated by the observation that, over a period of one-and-a-half years, more than 500 new papers on graph theory and its applications had to be added in updating an indexed worldwide bibliography on graph theory.[44] Although workers in the United States, Canada, and countries of western Europe have been most active in this area, the Soviet Union and eastern European countries have evidenced increasing activity also.

To keep abreast of Soviet activity in graph theory one would have to survey continually a large number of periodicals, many of which are not easily accessible. Some of these appear in English translation, but often these translations appear a year or two after the original article or are of poor technical quality. Approximately 45 percent of the papers referenced in this bibliography appeared in journals that are regularly translated into English. Even though there has been an increase in the number of Western workers having reading capability in Russian, the fact remains that a large part of the Soviet work is simply ignored. It is extremely uncommon to see in papers by Western authors any references to Soviet papers on graph theory.

The purpose of this survey has been to make better known the work of Soviet authors in the field of graph theory and its applications. We have covered virtually all available Soviet work in this field and have attempted to correlate and compare it with Western work. Hopefully the availability of this survey will prevent duplication of work done in the

U.S.S.R, will make specific results and the techniques of Soviet workers more widely known, and will stimulate new ideas and further progress in Western activity in the field of graph theory and its applications.

B. Coverage and Scope

For purposes of this survey we have attempted to include all Soviet papers dealing wholely or partly with the subject of graph theory and its applications. Papers in Russian by non-Soviet authors have not been covered. We believe that the survey is virtually complete for papers dealing with graph theory as such. In the area of applications some dividing line must be drawn as to what constitutes an "application" of graph theory. As a rule, we have tried to exclude papers that employ graphs merely as models, with no theoretical content, but have been generous in including some marginal papers.

Almost all of the papers referenced here have appeared in periodicals, aperiodicals, and proceedings of conferences. As mentioned, in many cases the translations available are not of good quality. Usually the meaning of the original article can be figured out by a reader reasonably conversant with the subject matter, but in a few cases we found that reference to the original article was necessary.

C. Procedure

The survey was carried out by exhaustively examining the contents of more than 50 Soviet scientific and engineering journals, including those known to contain the major contributions on graph theory and its applications. Most of these journals were covered from about 1960 to June 1968. This bibliography was then augmented by a similar coverage of several Soviet conference proceedings and collections, and both Soviet and U.S. abstract and review journals, and by tracing back the references

cited by authors of all of the Soviet papers encountered. Only a few papers could not be obtained at all. Most of these were old and not often cited, and are probably not significant; the rest were from fairly recent collections and conference proceedings that are not yet available, at least outside of the Soviet Union. About 40 percent of the papers had to be translated, either because translations or detailed abstracts were not available, or because the quality of the existing translations or abstracts was inadequate. The preparation of the final bibliography involved much verification and cross-checking, in order to remove errors, ambiguities, and duplications, and to permit the citation of translations and abstracts whenever these were found to be available.

D. Form of the Survey

Section II of this report consists of a summary of the new technical results on graph theory that were found in our coverage of the Soviet literature, as well as conclusions that may be drawn from these results regarding the typical Soviet attitude toward the field, strong and weak areas of competence, and comparisons with Western work (both specifically and in general). Sections III and IV constitute the bulk of the report; they describe in detail the numerous technical results, organized under the various headings listed in the table of contents.

The main bibliography (A) is arranged alphabetically by author, then by year, and then alphabetically by title. Titles of papers are given in translated form, journals in transliterated Russian or Ukranian (the British Standard System of transliteration being employed, without the redundant diacritical marks: ', ", ˘) and collections in either form. When journal translations or translated reviews or abstracts are available, these are also cited. Each entry is indexed with a letter (initial letter of author's name) and a number. The second bibliography (B) lists the

titles, both abbreviated and complete, of all periodicals and aperiodicals appearing more than a few times in the main bibliography (A), including the titles, publishers, and years of publication of available translations. Each is indexed with a J-number. The final list of references (C) contains the non-Soviet items referred to in the text; it is indexed numerically.

E. Prior Surveys and Symposia of Soviet Work in Graph Theory

Three partial surveys of Soviet work in graph theory have appeared. Zykov[Z29,Z31] has presented very brief surveys of recent work of Soviet graph theorists at two international meetings on graph theory. Zykov[Z28] has also written a more comprehensive survey of worldwide research in graph theory through 1962, with an emphasis on Soviet contributions. This present work is the first attempt at carrying out a comprehensive survey of Soviet activity in the field of graph theory.

With the exception of Zykov,[Z27] Soviet graph theorists did not contribute papers at the international symposia on graph theory held in Smolenice in 1962[15 K37] and in Rome in 1966.[36] Abstracts of a few Soviet graph theory papers appear in the Proceedings of the International Congress of Mathematicians, which was held in Moscow in 1966.[K24,G16,L11,Z15]

A session for papers on graph theory was included at the Eighth All-Union Colloquium on General Algebra. Titles of papers presented at this meeting appear in the article by Strasdin.[S30] Very few graph-theory papers were presented at earlier colloquia of this annual series.

II SUMMARY AND CONCLUSIONS

A. Summary of Technical Progress

In general the level of Soviet activity in graph theory and its applications lags behind that of Western workers in both quality and quantity. However, there are numerous Soviet papers in this field that merit being made more widely known among Western workers.

The first Soviet paper dealing in part with graph theory was by Kudryavtsev[K36] in 1948, and the first Soviet paper devoted entirely to graph theory was written by Zykov[Z18] in 1949. In the years to follow, Soviet capability in graph theory can be attributed most directly to one individual, Zykov.* Not only is his own work of high quality, but many of the best Soviet papers conclude with an acknowledgment of Zykov's assistance to the author.

Another significant influence on Soviet work has been the Russian translation of Berge's well-known book on graph theory.[5] Whether or not one finds Berge's terminology most appropriate, it has had the happy effect of introducing a relatively uniform terminology in the Soviet literature, a statement that cannot be made about Western literature on graph theory. Most Soviet papers cite Berge's book as a reference. In addition, numerous Soviet papers have dealt with the solution of problems left open in Berge's book and with the extension of theorems stated in it.

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* This is in spite of Premier Kosygin's answer (Life, February 2, 1968, page 32B) to an interviewer's question that "you are exaggerating the role of an individual in the Soviet Union--in our country it is the collective that works."

This survey discusses both "pure" and "applied" work in graph theory. Generally speaking, the pure work is more original and of a higher quality than the applied, and has shown more independence of Western work. The strongest areas of Soviet graph theory have been

(1) Bounds on numerical indices associated with graphs,

(2) Properties of algebraic structures associated with graphs, and

(3) Operations on graphs.

This work is discussed in detail in Sec. III.

The applied work, with a few exceptions, has tended to imitate and in a few cases refine Western work, but has seldom set new directions. These contributions are discussed in Sec. IV.

Some areas where Soviet work is relatively weak or non-existent are

(1) Connectivity properties of graphs, such as those discussed in the book by Tutte,[45]

(2) Exact enumeration of graphs with prescribed properties,

(3) Matroid theory,

(4) Graph isomorphism testing,

(5) Graph coloring (with the exception of Vizing's work on edge-coloring problems), and

(6) The use of graphs for modeling in the social sciences.

B. Conclusions

This survey has shown that an outstanding competence in many areas of graph theory exists among Soviet workers. Western graph theorists would therefore do well to make themselves aware of this work.

This survey should bring Western workers up to date on Soviet activity by calling attention to those Soviet results that may be relevant to their own work. To remain continually in touch with Soviet activity in

graph theory, however, one must be prepared to follow a large number of journals and conference proceedings, because the Soviet literature on graph theory is scattered through the mathematical and engineering literature. Those journals that have published most of the Soviet graph theory papers are Refs. J11 (15%); J3, J4, J10, J19, J22, J30, J31, J35, J42, J46 and J47 (4% to 7% each).

Although Mathematical Reviews[J26] has improved its coverage of Soviet journals, its coverage of Soviet graph theory is still neither comprehensive nor prompt. Referativnyi Zhurnal Matematika[J34] naturally covers the Soviet graphical work more quickly and thoroughly, but it has the obvious disadvantage of being available only in Russian.

Both Western and Soviet mathematicians have apparently had difficulty in deciding how to classify graph theory as a branch of mathematics. Papers on graph theory were formerly grouped under Algebraic Topology in both the Soviet and U.S. review journals. From October 1965 to December 1966 these papers were included under Operations Research in the Soviet review journal. Currently both journals classify most papers on graph theory in the section entitled Combinatorial Analysis, and both have recently included a special subsection on Graph Theory within this section. Also, as mentioned in Sec. I-E, one of the areas of Soviet strength in graph theory is in the field of algebraic structures associated with graphs. Thus, one should survey those portions of the review journals that deal with algebra, groups and semigroups, in particular.

The following specific needs have been identified during the survey:

(1) In view of the poor technical quality of many English translations of Soviet papers on graph theory, translators should be employed who have a better technical knowledge of the field, as well as a good knowledge of the Russian language.

(2) Translations of numerous additional significant Soviet graph-theory papers should be made available. In particular graph theory abstracts from Referativnyi Zhurnal Matematika[J34] should be translated into English and published promptly, either in Mathematical Reviews[J26] or in some other journal.

(3) In the future, surveys for updating Soviet work in graph theory should be prepared at least every two or three years. The present survey could serve as a starting point for these later versions.

(4) Several Soviet papers have referred to a forthcoming book, The Theory of Finite Graphs, by A. A. Zykov. We recommend that this book be seriously considered for translation into English as soon as possible after it becomes available in the West.

III PURE GRAPH THEORY

The terminology used in this survey is for the most part that used by the Soviet authors themselves, which in turn is most often that used by Berge.[5] A graph G is usually designated in the form $G = (X,E)$, where X is the set of points of G and E is the set of edges. Digraphs are usually designated by $G = (X,\Gamma)$ or $G = (X,U)$, where Γ denotes the successor function, i.e., $\Gamma_x = \{y: (x,y)$ is an arc of $G\}$ and U denotes the set of arcs of G. A sequence of distinct edg... and distinct points where consecutive edges are incident is called a <u>path</u>. Closed paths are called <u>cycles</u> or <u>circuits</u>. <u>Multigraphs</u> are graphs or digraphs where cycles of length two, i.e. parallel edges, and cycles of length one, i.e. loops, can occur.

The <u>degree</u> of a point is the number of edges incident at the point. A graph is <u>regular</u> if the degrees of all its points are equal. The term <u>subgraph</u> is generally used to denote some subset of points of a graph together with all edges of the original graph that connect these points. The term <u>partial graph</u> of graph G designates a graph having all the points of G but only some of the edges of G. Most other definitions are included with the discussion of papers in which they are first used.

Although this chapter is reasonably self-contained, the authors assume that the reader is at least somewhat familiar with the elementary concepts of graph theory and elementary set theory.

A. <u>Graphical Parameters</u>

In studying many mathematical structures it is useful to be able to introduce certain parameters whose values give some insight into the global properties of the structure. Numerous Soviet papers deal wholly

or partly with this problem for graphs and digraphs. Definitions are given here for those parameters most often studied.

Let $G = (X,\Gamma)$ be a digraph. A subset of its vertices $S \subseteq X$ is called internally stable if $\Gamma S \cap S = \emptyset$, i.e. if no two vertices in S are adjacent. A set $T \subset X$ is called externally stable if $x \notin T$ implies $\Gamma x \cap T \neq \emptyset$, i.e. if x is not in T, then some vertex adjacent to x is in T. The coefficient of internal stability $\alpha(G)$ is the cardinality of a largest internally stable set of G. The coefficient of external stability $\beta(G)$ is the cardinality of a smallest externally stable set of G.

For a given graph $G = (X,E)$ the strong product graph $G^n = G \times \ldots \times G$ is defined as follows. The vertex set for G^n is the set of n-tuples of elements of X, i.e. the set X^n. The vertices $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ are adjacent in G^n if for all i either $x_i = y_i$ or $(x_i, y_i) \in E$. The capacity of a graph G is the number

$$\theta(G) = \sup_n \sqrt[n]{\alpha(G^n)} \quad .$$

In a graph the distance from x to y is the number of edges in a shortest path from x to y The diameter of a graph is the maximum distance between any two of its points. The elongation of a point x is the maximum distance of any point from x. The radius of a graph is the minimum elongation of any of its points.

For a graph $G = (X,E)$ with p points, m edges, and ℓ connected components the cyclomatic number $\nu(G)$ is given by

$$\nu(G) = m - p + \ell \quad .$$

The density of G is the cardinality of a largest complete subgraph of G.

The chromatic number $\chi(G)$ of a graph G is the minimum number of classes into which the points can be partitioned so that no two points of the same class are adjacent.

The chromatic index $q(G)$ of G is the least number of classes into which the edges can be partitioned so that no two edges of the same class are incident at the same point.

The genus $\gamma(G)$ of a graph is the lowest genus of a compact orientable 2-manifold in which the graph can be embedded. For example, the genus of any planar graph is zero.

Two Soviet papers by Lyubich[L22] and Mityushin[M26] deal with the capacity of a graph. Lyubich shows that the definition given for $\theta(G)$ can in fact be replaced by the definition

$$\theta(G) = \lim_{n\to\infty} \sqrt[n]{\alpha(G^n)} ,$$

i.e. the supremum can be replaced by a limit. Berge has already pointed out that this value is finite.[5]

From the definition one can easily see that $\theta(G) \geq \alpha(G)$. Berge[5] states mild conditions under which equality holds. Mityushin shows, however, that the ratio $\theta(G)/\alpha(G)$ can be made arbitrarily large as G varies. His method uses the following two lemmas.

Lemma 1 $\alpha(A \circ B) = \alpha(A) \cdot \alpha(B)$ where $A \circ B$ has vertex set $X_A \times X_B$ and a point (a,b) is adjacent to point (a',b') if and only if

(1) a is adjacent to a' or (2) $a = a'$ and b is adjacent to b'.

Lemma 2 $\alpha[(A \circ B) \times (A \circ B)] \geq \alpha(A \times A) \cdot \alpha(B \times B)$.

These lemmas are applied to the 5-cycle graph A_0 which satisfies $\alpha(A_0) = 2$, $\alpha(A_0 \times A_0) = 5$ and $A_k = A_{k-1} \circ A_{k-1}$.

Perhaps the three simplest parameters associated with a graph are the number of points, the number of edges, and the number of components. Several Soviet papers have dealt with problems of finding bounds for the more complex parameters of a graph in terms of these simple parameters.

Vizing[V26] has given estimates for the coefficient of external stability of a graph in terms of these parameters. His values for $\beta(G)$ satisfy

$$1 \le \beta(G) \le \begin{cases} \min\left\{\left[\frac{p}{2}\right], \left[\frac{1 + 2p - \sqrt{8m + 1}}{2}\right]\right\} & \text{if} \quad m \le \frac{(p-2)(p-3)}{2} \\ \min\left\{\left[\frac{p^2 - 2m}{2}\right], 2\right\} & \text{if } m > \frac{(p-2)(p-3)}{2} \end{cases},$$

where G is a connected graph with p points and m edges, and $[x]$ denotes the integer part of x.

If we do not insist that G be connected, then

$$\max\{p - m, 1\} \le \beta(G) \le [p + 1 - \sqrt{1 + 2m}] \quad .$$

These bounds are best possible in the sense that in the class of all p-point graphs having m edges there exists a graph with coefficient of external stability equal to any preassigned integer indicated by the bounds above (similarly for connected graphs).

Ershov and Kozhukhin[E7] have done similar work for the chromatic number of a graph. Their result is that for a connected graph G with p points and m lines, the chromatic number $\chi(G)$ satisfies

$$-\left[-\frac{p}{\left[\frac{p^2 - 2m}{p}\right]}\left(1 - \frac{\left\{\frac{p^2 - 2m}{p}\right\}}{1 + \left[\frac{p^2 - 2m}{p}\right]}\right)\right] \le \chi(G) \le \left[\frac{3 + \sqrt{9 + 8(m - p)}}{2}\right],$$

where $[x]$ and $\{x\}$ denote respectively the integral and fractional part of x. Again results are the best possible, since connected graphs with p points and m lines are constructed which take on the extreme values.

For a given number p of points one can ask the question of how many edges can be "filled in" without having the radius of the resulting graph drop below a prescribed value r. Vizing[V28] has answered this question as follows. Let $f(p,r)$ denote the maximum number of edges in a graph of p points having radius r.

<u>Theorem</u> $f(p,1) = \frac{p(p-1)}{2}$, $f(p,2) = \left[\frac{p(p-2)}{2}\right]$

$$f(p,r) = \frac{p^2 - 4pr + 5p + 4r^2 - 6r}{2} \quad \text{for} \quad r \geq 3 \quad .$$

A digraph $G = (X,\Gamma)$ is said to be <u>strongly connected</u> if given points x and y in X, there is a path in G from x to y. Berge[5] points out that for a strongly connected digraph of p points, m arcs, and diameter δ the following inequalities hold:

(1) $p \leq m$

(2) $m \leq p(p-1)$

(3) $\delta \leq p-1$.

Berge also points out that while no one of these inequalities can be sharpened, the system of them can be improved, i.e. there exist numbers p, m, and δ that satisfy (1), (2), and (3) but that are not the parameters of any strongly connected digraph. Goldberg[G11] derives a lower bound for the diameter of a strongly connected graph that "completes" this system of inequalities. His inequality is

$$\delta > \left\{\frac{2(p-1)}{m-p+1}\right\} ,$$

where in this case $\lceil x \rceil$ denotes the least integer greater than or equal to x. This result also settles a conjecture of Bratton that Berge mentions.

Goldberg[G8] has also considered questions dealing with the radius of a strongly connected digraph. A digraph G is said to be minimally connected if G is strongly connected, but this property is lost if any arc of G is removed. Let G be a strongly connected digraph (the results hold even if G has multiple arcs) with p points, m arcs, cyclomatic number $\nu = m - p + 1$, and radius ρ. Then

(1) If G is minimally connected, then $m \leq 2p - 2$.

(2) If $1 \leq t \leq \nu$, then G has at least $\nu - t + 1$ strongly connected subgraphs with cyclomatic number t.

(3) $\rho\nu \geq n - 1$.

Examples show that all these inequalities are sharp. The technique used in proving these inequalities is what Goldberg calls the "contraction operation," which consists of deleting all arcs of some subgraph H and identifying the points of H.

Vizing[V23,V24,V25] has done interesting work on edge colorings of graphs and multigraphs. Let G be an s-graph (i.e. two vertices may be joined by up to s edges) and let $\sigma(G)$ denote the maximum degree of any point of G.

Theorem If G is an s-graph and $q(G)$ denotes the chromatic index of G, then

$$q(G) \leq s + \sigma(G) .$$

Corollary If G is a graph, then

$$\sigma(G) \leq q(G) \leq \sigma(G) + 1 .$$

For $m \geq 2$ there exists a graph G such that $\sigma(G) = m$, $q(G) = m + 1$.

Vizing points out that for $s \leq [1/2\ \sigma(G)]$ this upper bound for $q(G)$ is better than Shannon's bound[40]

$$q(G) \leq [3/2\ \sigma(G)] \quad .$$

He also points out that the bound for $s > 2$ is not the best possible in the following sense. If $s > 2$ and $\sigma(G) = 2s + 1$, then $q(G) \leq \sigma(G) + s - 1$. However, if $\sigma(G) = 2ks$, $k \geq 1$, and G is a $(2k + 1)$-point s-graph in which every pair of vertices is connected by s edges, then $q(G) = \sigma(G) + s$. Thus a sharp upper bound for $q(G)$ for $s < [\sigma(G)/2]$ must depend on the relationship between $\sigma(G)$ and s. Vizing proposes the determination of this relationship as a subject for further study.

In Ref. V24 Vizing proves several theorems that give more detailed information on the chromatic index of special classes of graphs. Some typical examples are the following.

<u>Theorem</u> If G is a regular graph of even degree d with an odd number of points, then $q(G) = d + 1$.

<u>Theorem</u> $q(K_{2k+1}) = 2k + 1$, where K_ℓ denotes a complete graph on ℓ points.

<u>Theorem</u> $q(K_{2k}) = 2k - 1$

<u>Theorem</u> For integers $m \geq 3$, $k \geq 3$ there exists a graph G with $\sigma(G) = m$, $q(G) = m + 1$, $t(G) \geq k$, where $t(G)$ = girth of G—that is, the length of the shortest circuit of G.

In Ref. V25 a <u>critical graph</u> is a graph G having the properties that (a) G is connected, (b) $\sigma(G) + 1 = q(G)$, (c) by removing any edge of G its chromatic index is reduced to $\sigma(G)$. Several interesting theorems

are proven for these graphs, some problems are posed, and some conjectures are stated. Namely:

Theorem In a critical graph with $\sigma(G) = m$, each point adjacent to a point of degree k is adjacent to at least $m - k + 1$ points of degree m.

Theorem A critical graph with $\sigma(G) = m$ contains an elementary circuit of length $m + 1$.

An interesting problem here is to get a lower bound on the length of the longest elementary circuit in a critical graph of degree m that takes into account the number of points of G.

Theorem In a critical graph of degree m, the number of edges is at least $(\quad + 6m - 1)/8$.

Conjecture: The number of edges is at least m^2 2.

Let L_k denote the set of graphs such that $G \in L_k$ if and only if every subgraph of G contains a point of degree not exceeding k.

Theorem If $G \in L_k$ and $\sigma(G) \geq 2k$, then $q(G) = \sigma(G)$.

Theorem If G is a planar graph and $\sigma(G) \geq 8$, then $q(G) = \sigma(G)$.

In connection with this latter theorem, Vizing raises the following question. For $2 \leq m \leq 5$ it is easy to construct a planar graph with $\sigma(G) = m$ and $q(G) = m + 1$. What is the situation for $m = 6$ and $m = 7$?

There are relatively few Soviet papers on other aspects of graph coloring. Loginov[L12] proved that if the maximum degree of the vertices of a graph G is 3, then, with the exception of K_4, G has chromatic number at most 3. This is just a special case of Brooks' Theorem.[6] See Ref. 34 for a discussion of this theorem and other Western work on graph coloring.

Yaakson[Y1,Y2] has discussed edge colorings of planar graphs that are regular of degree 3. The colorings use only two colors p and q such that at every point one edge is colored p and the other two edges are colored q.

It is well known that a necessary and sufficient condition for the existence of a proper four-coloring of the faces of such a trivalent map is the existence of a two-coloring of the edges of the above type in which the cycles formed by the edges of color q all have even length. Yaakson gives some sufficient conditions for the existence of such colorings.

Dynkin and Uspenskii[D7] have written a book on coloring problems that should be accessible to students at the high school level. The book has been translated into German and English.

Yaglom[Y4] has surveyed some work of Ringel on the chromatic numbers of graphs drawn on closed surfaces of posi genus.

Some difficult unsolved problems in graph theory are concerned with finding *efficient* algorithms for the actual computation of graph parameters such as the chromatic number and coefficient of internal stability. Maghout[29] has given algorithms for solving these problems that involve the simplification of Boolean functions. However, all these procedures have the serious defect that the amount of computation required grows *exponentially* with the number of points of the graph. Much to be desired are algorithms for solving these problems that involve only *algebraic* growth of the required computation time. Soviet authors are well aware of the shortcomings of existent algorithms, but they have not been any more successful than Western workers in devising appreciably more efficient algorithms.

Vitaver[V19] proves the following theorem. Let G be a graph, $\chi(G)$ its chromatic number, and $k(G)$ the greatest integer m, such that every digraph obtained by directing the edges of G contains at least one path of length m.

Theorem $\chi(G) = k(G) + 1$

Vitaver then observes that this theorem can be used to compute $\chi(G)$ by computing Boolean powers of a variable adjacency matrix of G. The method would be inefficient and difficult to implement on a computer.

Vizing and Plesnevitch[V27] have "reduced" the problem of finding the chromatic number of a graph to the problem of finding the coefficient of internal stability of a certain product graph.

<u>Theorem</u> Let a graph G have n points and let K_p be the complete graph on p points. Then $\chi(G) \leq p$ if and only if $\alpha(G \times K_p) = n$. The product graph $G \times K_p$ has (x,y) adjacent to (x',y') if and only if (1) $x = x'$ and y is adjacent to y' or (2) $y = y'$ and x is adjacent to x', i.e. the Cartesian product of G and K_p.

<u>Corollary</u> $\chi(G) = \min \{p: \alpha(G \times K_p) = n\}$ where G has n points. The authors then give a procedure for calculating $\alpha(G \times K_p)$ by again "reducing" the problem to a problem of network flow theory. They associate with the given graph a certain network $T_{G'}$, and associate capacities with the arcs of this network in such a way that the following theorem holds.

<u>Theorem</u> The value of the maximum flow on $T_{G'}$ is $2p\alpha(G')$. Any maximum flow of G' on the net $T_{G'}$ determines a largest internally stable set of G' consisting of the beginning points of output arcs carrying 2p units of flow.

Grinberg and Ilzinia[G15] and Ilzinia[I1,I2,I3] have considered a graph coloring problem associated with coloring wires in network assemblies. The efficiency of their procedure is highly dependent on the initial numbering of the points of the graph. To improve efficiency they are forced to consider the secondary problem of finding the cliques of the graph, i.e. the maximal complete subgraphs of G. Thus as part of their coloring algorithm they also compute the density (or <u>clique number</u>) of the graph. The procedures given for carrying out the graph coloring involves a

certain amount of trial and error testing and do not appear to be highly efficient. It is worth observing that the calculation of the density and coefficient of internal stability of a graph are essentially equivalent problems, since the density of a graph G is equal to $\alpha(\bar{G})$, where $\bar{G}$ is the complement of G.

Osis[O2] gives two algorithms for finding minimal externally stable sets of a graph. One involves simplification of Boolean functions, and the other involves operations on the adjacency matrix of the graph. Neither of them appears to be particularly efficient.

Vakhovskii[V2] describes a method of decomposing a graph that in some cases can simplify the calculation of $\alpha(G)$. The decomposition technique proceeds only as long as there remains at least one point in the graph of degree 1. Each step of the decomposition (1) locates a point p of degree 1 whose adjacent point is, say, q and (2) removes the points p and q and all edges incident at point q. The process stops when no points of degree 1 remain. G is <u>completely decomposable</u> if the process yields a graph with only isolated points. The following theorems are proven.

<u>Theorem</u> As a result of the decomposition we obtain the same number of isolated points and the same indecomposable part G_1 of G independent of the order of decomposition.

<u>Theorem</u> If E denotes the set of "first" points of edges incident upon a point of degree 1 that is removed in the decomposition process, N the remaining isolated vertices, and P a maximum internally stable set of points of the indecomposable part of G, then $K = E \cup N \cup P$ is a maximum internally stable set of points of G.

A <u>matching</u> of a graph is a subset of its edges no two of which are incident at the same point.

Theorem If W_1 is the set of "first" edges removed in a decomposition of G and W_2 is a maximum matching of the indecomposable part of G, then $W_1 \cup W_2$ is a maximum matching of G.

These latter two theorems simplify the process of finding maximum internally stable sets of vertices and maximum matchings of a graph. Of course they are only helpful when the graph G contains at least one point of degree 1. (The general problem of finding maximum matchings has been solved efficiently by Edmonds[13] and is one of the relatively few problems of this type for which efficient algorithms are known.)

Vetukhnovskii[V11,V12,V13] has considered coverings of a graph by neighborhoods of its points where these neighborhoods have prescribed radii. A spherical neighborhood of radius r of a point a of a graph G consists of all points of G within distance r of a. A graph is covered by a system Σ of neighborhoods of some of its points if all points of G lie within some element of Σ. Similar definitions describe a covering of the edges of G.

Theorem Let $\Sigma_0(\Sigma_1)$ be a point (edge) covering of the connected graph G. Then there exists a spanning tree $D_0(D_1)$ of G such that the system $\Sigma_0(\Sigma_1)$ is a point (edge) covering of $D_0(D_1)$.

Vetukhnovskii then describes a procedure for finding a point covering of a tree where all the neighborhoods have equal radii and the covering is optimal with respect to minimizing the sum of the radii of a covering (using spheres of prescribed radius). He leaves this as an open problem for graphs that are not trees because, even though the previous theorem guarantees the existence of a spanning tree in the graph for which simultaneously optimal point coverings could be found, it does not tell how to find this tree. The paper also contains certain estimates of the complexity of coverings for several classes of graphs and

for various measures of complexity. See Ref. 23, pp 169-181, for a discussion of n-covers and n-bases of a digraph that is related to Vetukhnevskii's considerations for the case of equal radii.

A few other Soviet papers make at least some use of the parameters mentioned here, but they are more appropriately covered in other sections of this paper.

We conclude this section by commenting on a rather tantalizing abstract of a Soviet graph theory paper that appeared recently. Khomenko[K24] gives the correct formula for the genus of the complete graph

$$\gamma(K_n) = \left\{ \frac{(n-3)(n-4)}{12} \right\}$$

where $\{x\}$ denotes the least integer greater than or equal to x. No proof is given, of course, in the short abstract referred to. The result is indeed noteworthy because, as Youngs[51] has pointed out, the above formula for $\gamma(K_n)$ is equivalent to the truth of the Heawood conjecture. The latter conjecture states that the chromatic number of a compact orientable 2-manifold is given by the so-called Heawood number

$$H(\gamma) = \left[\frac{7 + \sqrt{1 + 48\gamma}}{2} \right] \quad \text{for} \quad \gamma > 0 \quad .$$

The Heawood conjecture was established only la[illegible]967 by Ringel and Youngs and has not even yet been published in detail. No detailed work by Khomenko has appeared on this question, so priority for resolving the Heawood conjecture must reside with Ringel and Youngs pending further information.

B. Digraphs

Although papers dealing with digraphs are discussed in almost all sections of this survey, this particular section deals with Soviet graph theory papers in which the fact that the graph is directed is of fundamental importance in the paper. Relatively little Soviet work falls in this category. The most comprehensive Western work on digraphs is the book by Harary, Norman and Cartwright.[23]

Barzdyn, Dambit, and Markosyan have written papers dealing with bases of digraphs.

A point basis of a digraph $G = (X,\Gamma)$ is a subset B of points such that (1) for all $p \in X$, $p \notin B$, there exists a path in G passing from some point of B to p, (2) no two points of B can be joined by a path of G.

A basis of arcs B is a set of arcs such that (1) if $(p,q) \notin B$ then there exists arcs in B forming a path from p to q (2) if $(p,q) \in B$ then no other path of branches in B goes from p to q.

Barzdyn[B3] has proven the following theorems, most of them concerned with the uniqueness of point bases and arc bases of a given digraph.

Theorem A digraph G has two point bases if and only if it contains a cycle and does not contain any point from which an arc passes to a point of this cycle.

Theorem A digraph G has two different arc bases if and only if it possesses at least one of the following properties:

(1) There exists in G two arcs (p,p') and (q,q') such that the points p and q belong to one cycle and p' and q' belong to a disjoint cycle such that any path that passes from any point of the first cycle to any point of the second cycle passes only through points of these cycles.

(2) There exist in G two arcs (p,r) [or (r,p)] and (q,r) [or (r,q)], such that the points p and q belong to one cycle and such that any path that passes from any point of this cycle to point r (or from point r to any point of this cycle) passes only through points of this cycle.

Other sections of Barzdyn's paper deal with the relation between arc bases of a digraph G and arc bases of the transitive closure of G. (The transitive closure $\hat{G}$ of G has the same point set as G, and if (p,q) and (q,r) are arcs of G then (p,r) is an arc of $\hat{G}$.) A final section deals with the addition of ideal points to a digraph. A digraph G^s is called an <u>expansion of G by the point s</u> if

(1) Points of G^s = points of $G \cup \{s\}$

(2) G is a subgraph of G^s

(3) In G^s there exist paths between only those points of G for which there exists a path in G.

The point s is called an <u>ideal point</u> for G if there exists an extended graph G^s such that an arc basis of G^s has fewer arcs than any arc basis of G. Barzdyn then proves several theorems related to this concept.

Dambit[D1] deals with the concept of a <u>strong point basis</u>. A point basis B of a digraph G = (X,Γ) is called <u>strong</u> if $b \in B$ implies $\Gamma b = \emptyset$.

<u>Theorem</u> The point basis B of G is a strong basis if and only if every point of B belongs to no circuit.

He gives several theorems that give necessary and sufficient conditions for the characteristic function of a set B in order that it be a strong basis for a digraph. Finally, three algorithms are given for finding a strong basis if one exists.

Markosyan[M2] derives a set of conditions for the uniqueness of basis of arcs of a digraph.* Recently Markosyan[M3] has given a procedure for finding a basis of arcs of a digraph by performing Boolean operations on certain matrices related to the adjacency matrix of the digraph. Criteria for uniqueness of the basis of arcs are given in terms of these matrices.

Grinberg and Dambit[G19] consider the problem of locating a minimal set of arcs in a digraph whose removal results in an acyclic digraph. If $G = (X,U)$ denotes the digraph, then minimal sets of arcs V such that $G' = (X,U-V)$ is acyclic are just those subsets W of U such that $G'' = (X,(U-W) \cup W')$ is acyclic where W' is formed from W by reversing the direction of arcs in W. A method for finding these minimal sets is given.

Lundina[L15] has extended a theorem of Rado on infinite digraphs. Let E_o be a finite set of integers. Associate with each subject $I \subseteq E_o$ a set $E(I) \subseteq E$. A function $\varphi: X \to E_o$ is a Rado function of the digraph $G = (X,\Gamma)$ if $\varphi(x) \in E(\varphi(\Gamma x))$ where $\varphi(\Gamma x) = \{\varphi(y) : y \in \Gamma x\}$. Rado has shown that if every finite subgraph of a locally finite digraph has a Rado function, then so does the graph itself. (G is locally finite if $|\Gamma x| < \infty$ for all $x \in X$. G is Γ-finite if $|\Gamma x| < \infty$ and $|\Gamma^{-1} x| < \infty$ for all x.) Lundina's result is the

Theorem Let $G = (X,\Gamma)$ be a Γ-finite digraph and $X = \bigcup_{i=1}^{\infty} X_i$ where (1) the X_i are finite, (2) $X_i \subseteq X_{i+1}$. If each subgraph $G_i = (X_i, \Gamma_i)$ has a Rado function φ_i then G has a Rado function.

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* Some confusion seems to exist between the papers of Barzdyn and Markosyan, because the former paper claims to give necessary and sufficient conditions for the uniqueness of the arc basis, whereas the latter paper claims that Barzdyn has given only necessary conditions.

Leifman and Petrova[L1] give an efficient algorithm for detecting whether closed paths have (mistakenly) been introduced in a PERT diagram. This problem arises in setting up project control diagrams, the graphs of which must be acyclic digraphs, but for which, due to their very large size, mistakes of the designer may introduce closed paths. Their procedure also locates those arcs that lie in closed paths.

Zadykhailo[Z2] has also given an algorithm for locating points of a digraph that lie on a circuit.

Leifman[L2] has given an efficient algorithm for finding the strongly connected components of a digraph. He points out that for large graphs the method of computing Boolean powers of the adjacency matrix of the digraph is not practical. His method proceeds more directly by removing sinks and sources and then considers reachability relations in the residual graph.

Agibalov[A4] gives an algorithm for finding all elementary paths from a point x to point y in a digraph.

Peter[P4] has considered transformations of Boolean expressions by associating what he calls a "Kantorovich graph" with different forms of the expression. These graphs are directed rooted trees.

C. The Construction of Graphs with Prescribed Properties

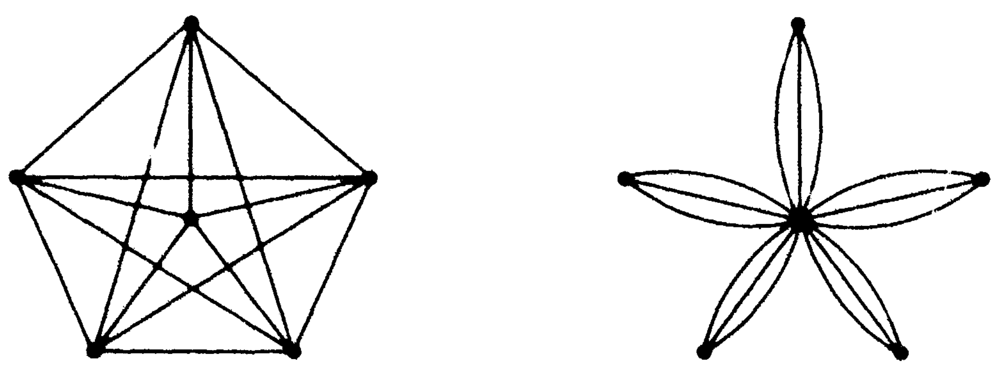
This section deals with the difficult graph theoretical problem of constructing graphs with certain prescribed properties. Naturally there is some overlap with other sections. For example, some of the papers described in Section III-A on finding bounds for the chromatic number of a graph in terms of other graph parameters also gave constructions of graphs that showed that the derived inequalities were sharp. Western work on these problems is so scattered through the literature that no particular attempt is made here to reference it.

Agakishieva[A3] treats two special cases of the following problem. Determine all graphs such that the subgraph generated by the points adjacent to any point of the graph is isomorphic to some prescribed graph H.

<u>Theorem</u> Let H be a path of length m. A necessary condition for the existence of a graph with every point having its neighborhood isomorphic to H is that $m \leq 7$. (She shows that $\ell = 2$ is impossible and gives examples for M = 1, 3, 4, 5, 6, 7).

<u>Theorem</u> Let H be a polygon of length m. A necessary condition for the existence of a graph with every point having its neighborhood isomorphic to H is that $m \leq 6$. (She gives examples for m = 3, 4, 5, 6.)

Kiryukin[K26] has treated the construction of communication nets of maximum reliability. Messages are to go from a single source to n receiving stations either directly or through a single intermediary station. Some fixed total number of channels (edges of the graph) are given, all having known probabilities of proper functioning. His main result is that under these conditions the structure of maximum reliability is one or the other of the types depicted in Fig. 1, the choice depending on a certain critical value of the retransmission probability.



(a) COMPLETE GRAPH WITH ONE SOURCE

(b) MULTIGRAPH WITH NO RETRANSMISSION

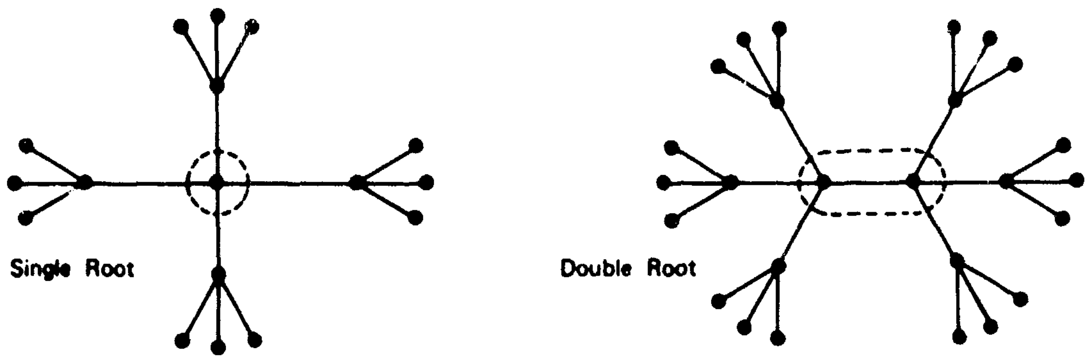
TA-6685-1

FIG. 1

Kelmans[K13] has considered the construction of graphs in which the edges have prescribed reliabilities. Let $R_q(G)$ denote the probability that G is connected if its edges have probability q of being present. A sequence of graphs $\{G_k\}$ is called absolutely reliable on $(q_0,1]$ if $\lim_{k\to\infty} R_q(G_k) = 1$ for $q \in (q_0,1]$. It is shown that complete graphs are absolutely reliable for any $q_0 \in [0,1]$. Other results show that the growth rate for the number of edges of a sequence of absolutely reliable graphs must be at least $\frac{p \ln p}{2\ln|1-q|}$ where p is the number of points of the graph. Sequences of graphs are constructed that are absolutely reliable and have near minimal number of edges. Finally, non-isomorphic graphs G_1 and G_2 with an equal number of points and edges are constructed such that for q near 0, $R_q(G_1) > R_q(G_2)$ and for q near 1, $R_q(G_1) < R_q(G_2)$. This shows that graphs with optimal reliability of being connected depend crucially on the factor q as well as on the structure of the graph.

This work is related to the work of Moore and Shannon[33] on constructing reliable relay networks from unreliable relays.

Zaslavskii[Z11] considers a graph construction problem that arose in studying models of human memory. A graph G has property (m,n) if the maximum degree of any point does not exceed m and the length of the longest path (with no repeated points) does not exceed n. G is called an (m,n) graph if it is maximal with respect to the (m,n) property, i.e. in any connected graph with a greater number of points the (m,n) property fails to hold. The author succeeds in characterizing (m,n) graphs as follows. For m = 2, n arbitrary, G is a path of length n. For $m \neq 2$, n even, the graph is depicted in Fig. 2(a) and for $m \neq 2$, n odd, the graph is depicted in Fig. 2(b).



(a) (m,n) GRAPH FOR m = n = 4

(b) (m,n) GRAPH FOR m = 4, n = 5

FIG. 2

Trufanov[T9] discusses briefly the problem of constructing graphs of minimum diameter with a prescribed number of points and edges. He treats in more detail two other questions. The first is to construct a class of infinite planar graphs all of whose faces are ℓ-gons with p edges incident at each point. For these graphs he derives values for the number of points that can be reached by a signal in a given length of time. The signal emanates from some distinguished point and covers one edge of the graph in one unit of time.

Secondly, he considers the problem of constructing finite regular graphs every point of which is a center. A center of a graph is a point having minimum elongation.) A class of such graphs is constructed for which bounds are given for their radii. The class is obtained by starting with a cycle and then drawing equal-length chords from all points of the cycle.

Sachs[38] has recently reported on work of the Soviet graph theorist Grinberg, who has constructed a planar graph that is regular of degree three, is cyclically 5-connected, has no Hamilton circuit, and contains 46 points. The simplest known example of a graph with these properties before Grinberg's example appeared was given by Walther[47] and had 114 points.

Reshetnyak[R3] considers a graph synthesis problem arising in the design of computer systems. The points of the graph represent "elementary" computers that are to be connected together to form a very-high-speed computer system. From each point of the graph x we want to be able to connect to an associated set of points f(x) by paths such that for $x_i \neq x_j$, the set of paths from x_i to points in $f(x_i)$ are <u>arc disjoint</u> from the set of paths connecting x_j to points in $f(x_j)$. The author calls f a <u>commutation of order m</u> if $|f(x)| \leq m$ for all x and $|f(x)| = m$ for some x. The associated graph is optimal if it minimizes the maximum degree of any node, is able to realize any commutation of order m, and has a minimum number of edges. Reshetnyak points out the difficulty of constructing such graphs and makes the following

<u>Conjecture</u>: The n-cube realizes any commutation of order 1, i.e. given any single-valued function f defined on the points of the n-cube, there exist paths L_i, i = 1, 2, ..., 2^n going from i to f(i) such that L_i and L_j are arc disjoint for $i \neq j$. The edges of the n-cube are regarded as bidirected arcs. The author has proven the conjecture only for n = 2 and tested it for numerous cases for n = 3, 4.

Some related Western work has been done by Watkins,[48] who requires that the paths joining the points are point-disjoint rather than just arc-disjoint.

Gorbatov[G13] has considered the problem of constructing digraphs of a type he calls "bottlenecks," which have prescribed circuit-arc matrices. A <u>bottleneck digraph</u> is one in which there exists a point through which all circuits pass and for which every path can be extended to be part of a circuit.

A few other Soviet papers deal at least in part with the construction of graphs with prescribed properties, but they are more appropriately discussed elsewhere.

D. Graph Isomorphism and the Coding of Graphs.

The papers covered here deal with finding efficient procedures for determining whether or not two graphs are isomorphic or, more generally, whether a given graph is isomorphic to a subgraph or a partial subgraph of another graph. Also, procedures for efficient coding of graphs are discussed. These problems are all difficult, and relatively little progress has been made on them in the Soviet literature.

Smolenskii[520] has given a procedure for coding trees by considering the distances between points of degree 1 of the tree (endpoints). If a tree T has s endpoints and ρ_{ij} denotes the distance from end point i to endpoint j, and then ρ_{ij} is called a metric for the tree.

Theorem Any real metric defines a single tree.

Using this theorem, Smolenskii points out that points of degree 2 or more can be located by observing that such a point must lie on a path between two endpoints and then specifying its distance from one of the endpoints. He makes some remarks about coding connected graphs that are not trees by finding a spanning tree of the graph, and somehow using this tree to code the graph. These remarks are not at all clear.

Zaretskii[78] extends the work of Smolenskii by giving necessary and sufficient conditions for a given matrix to be the matrix of distances between endpoints of a tree.

Theorem Let A be an n $\times$ n matrix. Necessary and sufficient conditions for the existence of a (unique) tree T with n endpoints such that a_{ij} is the distance from endpoint i to endpoint j are

(1) All a_{ij} are integers and $a_{ij} = a_{ji} > 0$ for $j \neq i$; $a_{ii} = 0$

(2) For all i, j, k the numbers $a_{ij} + a_{jk} - a_{ik}$ are even

(3) Among the numbers $a_{ij} + a_{k\ell}$, $a_{ik} + a_{j\ell}$, $a_{i\ell} + a_{jk}$, two are equal and not less than the third for all i, j, k, ℓ.

Kelmans[K15,K17] has rediscovered the results of Smolenskii and Zarteskii and extended them somewhat in the following theorem.

Theorem Let a tree T have s points of degree 1. This tree is uniquely determined by giving as coordinates for its points a collection of any s - 1 of its distances from the s points of degree 1. On the other hand, for any s a tree exists having s endpoints and having an internal point that is not uniquely determined by any set of s - 2 of its distances from the endpoints.

Related Western work on these problems has been done by Goldman,[19] Hakimi and Yau,[20] and Simoes-Pereira.[41]

Melikhov, Bershtein, and Karelin[M22] have proposed a method for testing isomorphism of digraphs. Their method involves computing the in-degree and out-degree of the points of the digraph and then making the observation that under an isomorphism, points of equal in-degree and out-degree must correspond. This simplifies somewhat testing for isomorphism, but of course would be no help at all for digraphs in which a large percentage of the points had equal in-degree and out-degree. The authors show how their graph isomorphism procedure can be used to test for the isomorphism of two automata by attempting to set up a correspondence between inputs so that the transformation graphs generated by corresponding inputs are isomorphic.

An interesting approach to the graph isomorphism problem has appeared in a recent Western paper by Corneil and Gotlieb.[11]

One of the most important practical problems in which graph and subgraph isomorphism problems arise is related to information search problems in chemistry. The structural formula of a chemical compound can be regarded as a labelled graph with the points corresponding to the various chemical elements and the valence bonds corresponding to edges of the graph. A common problem in chemistry is to locate all chemical compounds in a certain catalog whose structural formulas contain a prescribed substructure.

Borshchev and Rokhlin[B17] have proposed a method for recording a labelled graph in a computer memory in such a way that one can efficiently scan the stored graphs to see whether or not a given graph G is a subgraph of one of them.

Adelson-Velskii and Landis[A1] treat the problem of storing information without repetition. They seek to minimize the amount of search necessary to determine whether or not a new item has or has not appeared in the list to date. The entries are coded into a graphical tree structure as they arise. The information is stored so that the number of operations necessary to determine whether or not a new item has already appeared is at most $C\log_2(N+1)$, where C is constant and N is the number of items already stored. An algorithm is given for building up the tree.

Khizder[K21] has examined the information storage method of Adelson-Velskii and Landis and obtained approximate evaluations of the number of operations that take place in searching one of the trees.

Stotskii[S29] has also considered the relation between information storage and tree structures. He discusses procedures for coding rooted trees in two- and three-letter alphabets. In addition, his coding method enables one to derive bounds on the number of trees $D(n)$ have n edges. He shows that $D(n) < 4^n$.

A few Soviet papers deal with the relation between graph isomorphism and the isomorphism of certain algebraic structures associated with the graphs, but these are more appropriately discussed in Sec. III-G.

E. Enumeration Problems Related to Graphs

Relatively little Soviet work has been done on enumeration problems. A few papers deal with asymptotic values for the number of graphs of certain types, a few deal with exact counts of graphs of specified types, and the remaining ones deal with counting the number of subgraphs of specified types of a given graph, the number of colorings of a graph or numbers of paths joining points in a graph. For a survey of Western work in problems of graph enumeration, see the article by Harary.[22]

Lupanov[L16,L17] has given asymptotic estimates for the numbers of graphs of various types. Let $G(n)$ be the number of connected graphs with n edges. Then

(1) $$G(n) = \left(\frac{2}{e} \frac{n}{\ell n^2 n} \gamma(n)\right)^n .$$

where

$$\frac{2 \ell n\ \ell n\ n}{\ell n\ n} \lesssim \gamma(n) - 1 \lesssim \frac{4\ \ell n\ \ell n\ n}{\ell n\ n}$$

(2) The proportion of connected graphs in which the number of points k satisfies

$$\left|k - \frac{2n}{\ell n\ n}\right| > \frac{14\ n\ (\ell n\ \ell n\ n)^{1/2}}{(\ell n\ n)^{3/2}}$$

approaches 0 with increasing n.

He derives very similar bounds for the number of graphs with or without loops, with or without multiple edges, and which may or may not be connected. Finally, defining a network as a graph with a distinguished set of nodes, he proves that the same asymptotic values hold. The proofs are quite long and tedious.

Vetukhnovskii[V10] has also given bounds for the number of planar graphs with n edges. If G_n denotes this number, then $A^n < G_n < B^n$ where A and B are constants. Two plane realizations of a planar graph are called isotopic if (1) their edges and faces can be numbered such that faces having the same numbers are formed by edges having the same numbers, and (2) the edges entering into corresponding points occur in the same order (say clockwise).

The author then gives bounds for the maximum number of non-isotopic realization of a planar graph.

Vetukhnovskii[V9] has studied the problem of enumerating indecomposable 2-terminal networks. A network is indecomposable if it cannot be obtained by substituting a copy of one 2-terminal network for each edge of a second 2-terminal network. By a constructive procedure he has shown that

$$\varphi(n) > \left(\frac{n}{2e\ell n^2 n}\right)^2$$

where $\varphi(n)$ designates the number of these networks.

Yurtsun[Y7, Y8] points out that by using the Polya enumeration technique for calculating the number g_{np} of graphs of p points and n edges it is necessary to calculate the whole enumerating polynomial

$$g_p(x) = \sum_{n=1}^{p(p-1)/2} g_{np} x^n \quad .$$

He then gives a technique for deriving any one coefficient of this polynomial without working out all the terms. The method consists of regarding a graph as a word of length $p(p-1)/2$ in a two-letter alphabet and computing g_{np} as a sum over the cycle index of the full pair group on p points, and certain partitions of n. Yurtsun's second paper extends his results from two-letter alphabets to arbitrary alphabets.

Vinnichenko[V14,V15,V16,V17] has treated enumeration problems dealing with trees. In his first paper he gives the following formula for the number $Z''(p,\gamma)$ of rooted trees of height p and maximum degree γ.

$$Z''(p,\gamma) = \sum_{k=1}^{\gamma-1} \sum_{i=1}^{k} f(Z''(p-1,\gamma),i)\, S''(k-i) \quad ,$$

where $f(m,n)$ is the number of combinations with repetition of m things taken n at a time, and

$$S''(k-i) = \sum_{h_j,m_j} f(Z''(h_1,\gamma),m_1)\ldots f(Z''(h_j,\gamma),m_j) \quad ,$$

where the summation is over all integers m_j, h_j satisfying

(1) $$0 \le h_j \le p-2$$

(2) $$m_1 + m_2 + \ldots + m_j = k-i \quad ,$$

and $S''(0) = 1$. No numerical results are exhibited, and the formula does not appear to lend itself well to actual computation. These results are expanded in his second paper, where he derives generating functions whose coefficients give the numbers of trees of the type mentioned.

The author's third paper treats the problem of enumerating partially labelled rooted trees. The "labels" referred to here are certain abstract symbols. By suitable interpretation of the term "label," trees having prescribed properties can be enumerated. The exposition of the paper is very difficult to follow.

Since the formulas derived in the preceding papers are so complex, Vinnichenko has also considered approximate methods for counting trees. In Ref. V17 he gives some methods for obtaining approximate values for the number of trees of special types. The method involves an "extrapolation" procedures for later coefficients of the enumerating polynomial, assuming that the coefficients are known up to a certain value. Some numerical comparisons are made with exact values that show the power of the method.

Vinnichenko's most recent paper[V18] is concerned with enumerating what he calls "abelian words prescribed on a set of alphabets." In effecting this count he uses a graphical model that associates each point of a complete graph with one of the given alphabets, the point being weighted by the cardinality of the alphabet and the edges being weighted by the cardinality of the intersection of the corresponding alphabets.

Zakrevskii[Z5] considers the enumeration of state transition graphs of automata and specializes the problem to the enumeration of autonomous automata. This problem is equivalent to the enumeration of digraphs having the property that the number of arcs coming out of any point is equal to one--so-called functional digraphs or transformation graphs. Zakrevskii enumerates these graphs and also enumerates them for the special cases of (1) being connected and (2) being connected with the cycle having length one. Tables are given for $m \leq 25$ where m is the number of states, and diagrams are given that actually exhibit the graphs for small m. This problem was first solved by Harary.[21]

Dambit[D2] has given expressions for the number of spanning trees of a graph in terms of determinants formed from the fundamental circuit matrix L of the graph and the fundamental cut-set matrix of the gra. ı H. This value is given by any of the determinants $|L.L^T|$, $|H.H^T|$, $|{L \atop H}|$ where T denotes transpose. Some further results deal with spanning trees of planar graphs.

Kelmans,[K10,K12,K16,K17,K18] has written a series of papers dealing with the enumeration of spanning trees in a graph. He gives background motivation for why this is an important problem, and then proposes the following interesting approach to the problem. Select a set of operations on graphs from which one can build up larger classes of graphs. Find a set (preferably small) of characteristics of these graphs such that the number of trees in a graph derived by these operations is easily expressible in terms of the characteristics of the graphs on which the operations were performed. The operations he introduces are as follows. Addition of graphs $G_1 = (X_1, E_1)$ and $G_2 = (X_2, E_2)$ yields $G_3 = (X_1 \cup X_2, E_1 \cup E_2)$. Multiplication of G_1 and G_2 yields $G_3 = (X_1 \cup X_2, E_1 \cup E_2 \cup K_{v_i X_j})$ where $K_{v_i X_j}$ denotes all edges from point $v_i \in X_i$ to all points of $X_j - \{v_i\}$, $i \neq j$. Other operations are complementation, supersaturation (adding k edges between all pairs of nonadjacent points), and series connection (joining two graphs at a common point).

The following matrices and polynomials are introduced. The matrix $A = (\alpha_{ij})$ has $\alpha_{ii} = \deg(x_i)$, and $\alpha_{ij} = 1$ if x_i and x_j are adjacent, 0 otherwise.

The variable matrix V_r^p is obtained from A by (1) adding rI where I is the identity matrix and (2) setting some row equal to all 1's (p is the number of points in the graph). The author defines $B_r^p = \det(V_r^p)$ and points out that $B_r^p = \frac{1}{r}A(r)$ where $A(-r)$ is the usual characteristic polynomial of the matrix A. Kelmans calls B_r^p the <u>characteristic</u>

polynomial of the graph. The number of spanning trees of G is given by $D(G) = \frac{1}{p} B_0^p$. The author then works out relations for the functions B_r^p for the sum, product, etc. of nonintersecting graphs in terms of the B_r^p-functions of these graphs.

In the continuation of his first paper Kelmans used a result of Zykov[Z18] that says that a graph has a unique expression as a sum of products of elementary graphs. An elementary graph G is a connected graph whose complement is connected. Thus the problem of spanning-tree counting is reduced to tree counting in elementary graphs. He then computes the B_r^p function for a few special classes of graphs: (1) the one-point graph, (2) a chain, (3) cycles, (4) the series connection of two "stars," etc. These results are used to generalize previous work of Weinberg[49] and Bedrosian[4] on the number of trees in a graph.

In Ref. K18 Kelmans has studied in more detail the function B_r^p. Let $G = (X,E)$ be a graph, $Y \subset X$. G_Y denotes the graph obtained from G by collapsing Y to single point and deleting any loops that are formed.

Theorem If $B_r^p(G) = r^{p-1} + B_1 r^{p-2} + \dots + B_{p-1}$ is the characteristic polynomial of G then

$$B_k = \sum_{\substack{Y \subset X \\ |Y| = s-k}} D(G_Y), \quad k = 1, 2, \dots, p-1$$

where p is the number of points of G and $D(G_Y)$ is the number of spanning trees of G_Y.

Theorem Let $B_r^p(G) = \prod_{i=1}^{p-1} (r+r_i)$. Then (1) the r_i are real and non-negative, (2) $r_i \le \max_{i \ne j} \{d_i + d_j\}$ where d_i is the degree of point i, (3) if G has no multiple edges, then $r_i \le p$.

Several other theorems relate the B_i-values of a graph and its complement and discuss properties of the B_i for regular graphs. Two

final theorems of the paper discuss some classes of graphs for which these characteristic polynomials characterize the graphs up to isomorphism, and some other classes of graphs for which they do not characterize them up to isomorphism.

Kelmans[K11] has also shown that the percentage of unconnected labelled graphs of p points within the whole set of labelled graphs on p points goes to zero as $p \to \infty$, showing in an asymptotic sense that most graphs are connected. The proof is based on probabilistic considerations of connectivity properties of graphs in which the edges have prescribed probabilities of being broken. A procedure for recursively enumerating connected labelled graphs is given.

Geraskin[G1] has also treated the problem of finding trees of a network. Some discussion of computer calculation of these trees is given.

Kalninsh[K5] has considered the problem of enumerating edge colorings of multigraphs, all of whose components are multitrees. Two colorings are considered different if in one coloring there is a pair of edges with the same color and in the other they have different colors. Let the points of the graph be $x_1, \ldots, x_n$ with degrees $s_1, \ldots, s_n$ where $s_i \geq s_{i+1}$, $i = 1, \ldots, n-1$. Let c be the number of components of G and let p_j by the number of pairs of points linked by exactly j edges, $j = 1, \ldots, k$. Then

$$r_m(G) = \frac{\prod_{i=1}^{k} [(m-i)!]^{p_i} (m!)^{c-1}}{\prod_{i=1}^{n} (m-s_i)!} \quad \text{for } m \geq s_1 ,$$

where $r_m(G)$ is the number of colorings of G using exactly m colors. He points out that $r_m(G) = 0$ for $m < s_1$ and $m > \frac{1}{2} \sum_{i=1}^{n} s_i$ and gives for $r_m(G)$ for $s_1 \le m \le \frac{1}{2} \sum_{i=1}^{n} s_i$. The author also points out that $r_m(G)$ depends only on the number of components of G, the degrees of the points, and the numbers p_i. For more general graphs the structural features that determine $r_m(G)$ are not known.

Povarov[P11] has considered the problem of counting the number of paths between any two points of a graph (or digraph). Apparently this paper has been overlooked by Western writers. It gives an efficient procedure for solving the problem just mentioned, but in Ref. 23 the first solution of this problem is attributed to Parthasarathy,[35] whose paper appeared eight years after Povarov's paper. Povarov uses the adjacency matrix of the graph and introduces the concept of a quasiminor. Let $\|a_{ij}\|$ be a square matrix. Designate by $\|a_{ij,k\ell}\|$ the matrix obtained from it by deleting the k-th row and ℓ-th column, not changing the numbering of the remaining rows and columns. A quasiminor $|a_{ij,\ell k}|_{k\ell}$ where $k \neq \ell$ is the sum $\Sigma a_{ki_1} a_{i_1 i_2} \ldots a_{i_r \ell}$ where the summation is taken over all distributions $i_1 i_2 \ldots i_r$ ($r = 0, 1, \ldots, p-2$) of the number of terms of the matrix $\|a_{ij,kk,\ell\ell}\|$. For $k = \ell$ we let $|a_{ij,\ell k}|_{k\ell} = a_{kk}$. Here p is the size of the original matrix.

<u>Theorem</u> $A_{k\ell} = |a_{ij,\ell k}|_{k\ell}$ where $A_{k\ell}$ denotes the number of distinct paths from point k to point ℓ in a graph whose adjacency matrix is $\|a_{ij}\|$.

Pavorov gives a procedure for the calculation of quasiminors that is analogous to the expansion of a determinant by cofactors. The technique is embodied in the following theorem.

Theorem If $k \neq \ell$ then $|a_{ij,\ell k}|_{k\ell} = \sum_{m \neq k} a_{km}A_{km}^{(\ell)}$ where $A_{km}^{(\ell)} = 1$ for $m = \ell$ and $A_{km}^{(\ell)} = |a_{ij,\ell k}|_{m\ell}$ for $m \neq \ell$.

The procedure for calculating quasiminors could easily be implemented on a computer. Povarov's method also gives the number of paths of specified lengths between points, and can even give the number of closed paths passing through a given point.

One final general comment for papers of this section is that while the exposition in most of the Soviet papers on graph theory is fairly clear, the papers on enumeration are not so clear. In many cases it is difficult to tell what the author is trying to do and what his results mean.

F. Embedding Problems for Graphs

This section treats questions of two types. The first concerns the problem of determining whether a given graph is planar. Papers of the second type deal with questions of either embedding a given graph in a graph of some special type or embedding metrics in graphs of a certain type.

Plesnevich[P6] has given an algorithm for testing for planarity of a graph G. His procedure examines an elementary cycle μ and constructs an associated graph R_μ which, if G is planar, is bicolorable. If R_μ is not bicolorable, then by a lemma of the author, G is nonplanar. Operations are repetitively performed on the cycle, generating new cycles M until either (a) some R_μ is not bicolorable or (b) each R_μ is bicolorable and every point of G belongs to some cycle of the set M. In the latter case G is planar and in the former it is nonplanar. The algorithm may be modified to give the cyclic order of the points adjacent to each point in a plane realization of G.

Dambit[D3] has also given a necessary and sufficient condition for a graph to be planar. The criteria are given in terms of the incidence, cutset, and circuit matrices of the graph. It is also shown how a planar graph given by its incidence matrix can be embedded in the plane.

It would be interesting and worthwhile to make a more detailed comparison of the Soviet work on this question with the algorithms given by Western writers on this subject, for example, the work by Lempel, Even, and Cederbaum.[28]

Firsov[F2] has considered a graphical problem arising from the problem of deciding when two "linguistic objects" are "close." A graph G is said to be <u>matched</u> into an n-cube Q_n if the points of G are mapped into the points of Q_n such that there exists a monotone increasing integer-valued function f with $f(0) = 0$ and $f(\rho_{ij}) = r_{ij}$ where ρ_{ij} denotes the distance between points x_i and x_j in G and r_{ij} denotes the distance between the points of Q_n that correspond to x_i and x_j.

<u>Theorem</u> If each block of a graph can be matched (or isometrically embedded) in an n-cube, then the whole graph can be matched also.

<u>Corollary</u> Any tree can be matched into some cube of sufficiently large dimension.

<u>Theorem</u> Given any transformation f of the metric, there exists a graph not isometrically embeddable by means of f in a cube of any dimension.

Tylkin[T14] has considered the problem of determining whether a given $m \times m$ matrix of nonnegative integers can be the distance matrix of a set of m points on an n-dimensional cube. He gives necessary conditions that

for m ≤ 5 are also sufficient. He also gives bounds for the possible values of n in terms of m and the entries of the matrix. The results have application in coding theory.

Stotskii[S28] has generalized Tylkin's work in the direction of seeking embeddings of a given metric in some graph. A metric ρ_1 on a set X_1 is isomorphic to ρ_2 on X_2 if there exists a one-one correspondence $\varphi: X_1 \to X_2$ such that $\rho_1(x,y) = \rho_2(\varphi(x),\varphi(y))$, for all $x,y \in X_1$. A metric ρ is realized in a graph $G = (X,E)$ if there exists a subset $X' \subseteq X$ such that ρ is isomorphic to the natural graph metric on the subgraph generated by X'. ρ is realized on a graph $G = (X,E)$ if ρ is realized in G and (b) no edges may be removed from G so that ρ is realized in the resulting graph.

Theorem If ρ is realized in a graph G, then $d_\rho \leq d_G \leq 2d_\rho$, where d_G denotes the diameter of G and d_ρ denotes the diameter of ρ.

Theorem A metric ρ realized in a tree T cannot be realized with a smaller number of points or edges than there are in T.

Some other results give conditions under which uniqueness of the graphs realizing a metric holds. Other results pertain to lengths of circuits in graphs that realize a given metric.

Ziman[Z17] considered properties of those graphs that can be imbedded in the plane with integer coordinates as points and with nearest neighbor connections, i.e. a point can be adjacent to only the eight points nearest to it. He gives bounds for the chromatic number, chromatic index, and radii of such graphs.

G. Algebraic and Set-Theoretic Problems Associated with Graphs

Some papers in this section deal with the relations between properties of a graph and properties of certain algebraic structures associated with the graph. The most commonly studied such structure is the automorphism group of the graph. Davydov[D4,D5,D6] has done interesting work on questions related to this particular algebraic structure. Frucht[17] has shown that, given any abstract group G, there exist infinitely many graphs G whose automorphism groups A(G) are isomorphic to G. The problem of characterizing all graphs whose automorphism group is isomorphic to a given abstract group is a difficult unsolved problem. Davydov considers the somewhat more tractable problem of characterizing all graphs whose automorphism groups contain a given group as a subgroup. He treats this problem for several special cases of the chosen subgroup.

Consider first the case of the group generated by a single permutation g whose decomposition into cycles is of the form $g = (a_1 \dots a_{k_1}) \dots (a_{k_s} \dots a_{k_n})$. In terms of the adjacency matrices of the graphs (or digraphs), the problem then becomes that of determining all (0,1) matrices A that commute with the block diagonal matrix $T = (T_1, \dots, T_q, 1, \dots, 1)$ where T_i has the form

$$T_i = \begin{pmatrix} 0 & 1 & & & \bigcirc \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ 1 & \bigcirc & & & 0 \end{pmatrix}$$

and the sizes of the blocks are respectively $k_1, k_2, \dots, k_n$.

<u>Theorem</u> Let T be as above with ℓ the number of 1×1 blocks of T. Then for any matrix A, T commutes with A if and only if A can be partitioned into $(q+\ell)^2$ rectangular blocks A_{ij} of size $k_i \times k_j$ and the blocks A_{ij} are of the form

$$A_{ij} = \begin{pmatrix} B^{i,j} & \cdots & B^{i,j} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ B^{i,j} & \cdots & B^{i,j} \end{pmatrix}$$

where $B^{i,j}$ is a square s X s matrix with $s = g\ c\ d\ (k_i, k_j)$ and $B^{i,j} = a_0^{i,j} I + a_1^{i,j} P + \ldots + a_{s-1}^{i,j} P^{s-1}$ with P having the same form as the T_i.

By filling in the parameters $a_k^{i,j}$ that appear in the expression for $B^{i,j}$ with zeroes and ones, all adjacency matrices commuting with T, i.e., all graphs having g as an automorphism, are obtained.

He examines other special forms for the T matrix and determines properties of the adjacency matrices that commute with them. For example, T's of the block-diagonal form $T = (T_1^{i_1}, \ldots, T_k^{i_k})$ are considered where T_i is as before and the i_k assume integral non-negative values.

As a final result he shows that for graphs G with a prime number of points p and an automorphism group that acts transitively on the points of G, the adjacency matrix A of G can be expressed in the form $A = a_0 I + a_1 P + \ldots + a_{p-1} P^{p-1}$ where P is as before. This result is implicitly contained in papers by Chao[7] and Turner.[43]

In Ref. D6 Davydov associates a sequence of graphs with the automorphism group of a given graph as follows: Let A be the automorphism group of a graph $G = (X,E)$ and let $g_i, g_j \in A$. Define the distance between g_i and g_j to be $d(g_i, g_j) = \max_{v_k \in X} \rho(g_i v_k, g_j v_k)$.

The author proves that d is a metric on A. Construct new graphs B_k as follows. The points of B_k are the elements of A. Points g_i and g_j are adjacent in B_k if and only if $d(g_i, g_j) \leq k$. Let $B_k^{(i)}$ denote the

components of B_k with $B_k^{(1)}$ the component of the identity of A, and let $A_k^{(i)}$ denote the elements of A in component $B_k^{(i)}$.

Theorem $A_k^{(1)}$ is a normal subgroup of A for all k. The A_k are cosets of A with respect to $A_k^{(1)}$.

Corollary $A_0^{(1)} \subset A_1^{(1)} \subset \ldots \subset A_r^{(1)} = A$ is a normal series for A.

Corollary If the automorphism group A is simple, then all of its transformations have the same norm. (The norm of $g_i \in A$ is given by $N(g_i) = d(g_i, e)$ where e is the identity of A.)

Since infinitely many non-isomorphic graphs may have isomorphic automorphism groups, it is reasonable to attempt to associate some kind of algebraic structure with a graph that would more nearly characterize the graph. Virtually the only work done on this problem has been done by Soviet authors.

The first such candidate considered for the appropriate algebraic structure is the semigroup of endomorphisms of the graph. An endomorphism of a digraph $G = (X,E)$ is a mapping $f: X \rightarrow X$ such that $(x,y) \in E \rightarrow (f(x),f(y)) \in E$.

Gluskin[G6] studied the problem in a slightly different context; namely he considers a binary relation ρ on a set Ω. Let $S_\rho(\Omega)$ denote the semigroup of endomorphisms of Ω. A one-one mapping φ from a set Ω with relation ρ to a set Ω' with relation ρ' is called an isomorphism if $(x,y) \in \rho \Leftrightarrow (\varphi(x),\varphi(y)) \in \rho'$ and is called an anti-isomorphism if $(x,y) \in \rho \Leftrightarrow (\varphi(y),\varphi(x)) \in \rho'$ for all x,y. A quasi-ordering relation ρ is a relation that is reflexive and transitive.

Theorem Let ρ be a nontrivial quasi-ordering relation on the set Ω and let ρ' be a reflexive relation on Ω'. The semigroups $S_\rho(\Omega)$ and $S_{\rho'}(\Omega')$ are isomorphic if and only if the sets Ω and Ω' are isomorphic or anti-isomorphic. Every isomorphism of the semigroups $S_\rho(\Omega)$ and

$S_{\rho'}(\Omega)$ has the form $\varphi(x) = f\,x\,f^{-1}$ where $x \in S_{\rho}(\Omega)$ and f is an isomorphism or anti-isomorphism of Ω and Ω'.

In the context of graph theory however, the condition that the relation ρ be reflexive and transitive is unduly restrictive. Shneperman[S10] has given an example of a set Ω having two reflexive but nontransitive relations ρ and ρ' such that $\rho \subset \rho'$, $\rho \neq \rho'$ and $S_{\rho}(\Omega) = S_{\rho'}(\Omega)$. Thus one must seek even further for algebraic structures that characterize the corresponding graphs up to isomorphism.

Kalmanovich[K2, K3, K4] has attacked this problem by considering semigroups of partial endomorphisms of the graphs. A partial endomorphism of a graph $G = (X,E)$ is a partial mapping φ: $X \to X$ such that $(x,y) \in E$, $x, y \in \text{Domain}(\varphi) \to (\varphi(x), \varphi(y)) \in E$. Kalmanovich introduces three semigroups of partial endomorphisms of a digraph, namely (1) those mappings where φ is multiple valued (2) those mappings where φ is single valued, and (3) those mappings where φ is one-one. Let $S(X,\Gamma)$ be one of these semigroups corresponding to a digraph $G = (X,\Gamma)$.

Let X be a set, Γ a binary relation on X. Define $X_1 = \{x: x \in X, (x,x) \in \Gamma\}$, $X_2 = X - X_1$, $\Gamma_{ij} = \Gamma \cap (X_i \times X_j)$. Graphs (X,Γ) and (X,Γ') are conjugate if one of the following holds: (1) $\Gamma = \Gamma'$, (2) $\Gamma_{11} = \Gamma_{11}^{-1}$, $\Gamma_{22} = \Gamma_{12} = \emptyset$, $\Gamma' = \Gamma \cup \Gamma^{-1}$, (3) $\Gamma_{11} = X_1 \times X_1$, $\Gamma_{22} = \Gamma_{22}^{-1}$, $\Gamma_{12} = X_1 \times X_2$, $\Gamma' = \Gamma_{11} \cup \Gamma_{21} \cup \Gamma_{21}^{-1}$.

Theorem Let $G = (X,\Gamma)$ be a digraph that is neither reflexive nor irreflexive, $\Gamma \notin K(X)$, and $G' = (X',\Gamma')$ an arbitrary graph. Then the semigroups $S(X,\Gamma)$ and $S(X'\Gamma')$ are isomorphic if and only if the graph (X',Γ') is isomorphic to one of the graphs conjugate to the graph (X,Γ) or (X,Γ^{-1}).

(Here $K(X)$ consists of a few special relations on X that are much like universal relations on X.)

Thus this theorem shows that any of these three semigroups characterizes a large class of graphs up to the relation of conjugacy.

This still leaves open the problem of completing the characterization for the class of graphs not covered by the above conditions. Several other theorems and corollaries are proven in Ref. K2 that relate graph isomorphism questions to the isomorphism of certain semigroups.

In his most recent paper[K4] Kalmanovich has extended results of Gluskin on densely embedded ideals in semigroups of partial transformations. An abstract characterization of the semigroup of one-one partial endomorphisms of a graph is given by means of its densely embedded ideal.

Ivanova[I4] has studied isomorphism questions related to unary algebras. A unary algebra is a set with a unary operation defined on it. Thus the study of these algebras can be regarded as a study of digraphs in which each point has out-degree 1. Let A be a unary algebra and let A^n denote the direct product of n copies of A. (The direct product of two graphs $A_1 = (X_1,U_1)$ and $A_2 = (X_2,U_2)$ has point set $X_1 \times X_2$ with (x,y) adjacent to (x',y') if $(x,x') \in U_1$, $(y,y') \in U_2$.) In this paper Ivanova extends a result of Marica and Bryant[30] that says that for A and B finite, $A^2 \sim B^2 \rightarrow A \sim B$ where $A \sim B$ means A is isomorphic to B.

Theorem If A and B are finite and $A^n \sim B^n$, then $A \sim B$.

Some of Ivanova's results extend to the case of infinite unary algebras.

Theorem Let A have finite components and for each $h \geq 0$ the number of components of height h is finite. Then $A^n \sim B^n \rightarrow A \sim B$. (The height of a component is the least integer such that for all $x \in A$, x^n is a cyclic element. A cyclic element y satisfies $y^m = y$ for some $m > 0$.)

Some examples of infinite A and B are given such that $A^2 \sim B^2$ but $A \not\sim B$, e.g. let A have a countable number of one-cycles and a single two-cycle. B has a countable number of one cycles and two two-cycles. Then $A^2 \sim B^2$ but $A \not\sim B$. Other examples and theorems are given.

Shevrin and Filippov[S6] study partially ordered sets and their comparison graphs. Let $< P, \leq >$ be a partially ordered set. Its comparison graph is the set $< P, \sigma >$ where σ is the binary relation on P defined by $x \sigma y \Leftrightarrow x \leq y$ or $y \leq x$. The authors announce theorems that (1) give an abstract characterization of comparison graphs and (2) determine all partially ordered sets σ-isomorphic to a given graph $< P, \sigma >$.

Goldberg[G10] has studied a type of equivalence relations on digraphs that he calls stable equivalences. Let $G = (X, \Gamma)$ be a digraph. An equivalence τ is called <u>stable</u> if $x_1 \equiv x_2 \pmod{\tau}$, $(x_1, y_1) \epsilon \Gamma$, $(x_2, y_2) \epsilon \Gamma \Rightarrow y_1 \equiv y_2 \pmod{\tau}$. A <u>chain</u> $\mu = (\mu_1, \mu_2, \ldots, \mu_r)$ is a sequence of arcs of G such that μ_i and μ_{i+1} have a common endpoint but are not necessarily traced in the direction of their orientation. Let r' be the number of arcs of μ whose directions coincide with the direction of μ. Thus number $s = r' - (r - r')$ is called the pseudolength of μ. The index of a graph is the gcd of the pseudolengths of its cycles.

<u>Theorem</u> If in G each connected component has a cycle whose pseudolength is not zero, then the set of stable equivalences on G forms a lattice.

Using the concept of stable equivalences, Goldberg derives a formula for the number of connected components in a product of certain types of graphs, extending a result of McAndrew.[32]

<u>Theorem</u> If G and H are connected graphs each of whose points is the endpoint of some arc, then the number of connected components of the graph $G \times H$ is the gcd of the index of G and the index of H.

Shedivy[S5] has suggested studying the theory of relations from a graph theoretical viewpoint.

H. Operations on Graphs

This section deals with (1) ways of combining graphs and (2) graph properties that are recursively calculable by performing certain decompositions of the graphs. This has been a particularly active area of graphical research among Soviet workers. Papers discussed in this section have a fair amount of overlap with topics discussed in other sections.

Zykov,[Z18] in what is apparently the first Soviet paper on graph theory, treats the problem of combining graphs in sum and product operations that were defined in Section [illegible] in the discussion of Kelmans's papers. A graph G is elementary if both G and its complement $\bar{G}$ are connected.

Theorem Every graph G has a unique (up to order of the factors) representation as a product of sums of elementary graphs.

Theorem $\chi(G_1 + \ldots + G_n) \leq \chi(G_1) \cdot \chi(G_2) \ldots \chi(G_n)$, where $\chi(G)$ is the chromatic number of G, and examples show that the bound is sharp.

Zykov then gives a procedure for calculating the chromatic polynomial of a graph by adding edges and identifying nonadjacent points. The i-th coefficient of this polynomial gives the number of distinct colorings of the graph in i colors. Formulas for the chromatic polynomial of a sum and product of disjoint graphs G_1 and G_2 are given in terms of the chromatic polynomials of G_1 and G_2. Note the relation between this technique and Kelmans's techniques for counting spanning trees of a graph.

The next chapter of Zykov's paper is concerned with the density of a given graph. It introduces polynomials called dimensional polynomials whose coefficients give the number of complete subgraphs of each size in the given graph. Again formulas for the dimensional polynomial of a sum and product of disjoint graphs are given in terms of the corresponding

polynomials of G_1 and G_2. The following theorem shows that there exist graphs for which the chromatic number can be arbitrarily larger than the density.

Theorem To any two natural numbers χ_0 and d_0 with $\chi_0 \geq d_0 > 1$, there exists a graph G such that $\chi(G) = \chi_0$ and $d(G) = d_0$.

The last chapter of the paper considers graphs of special types. A graph G is said to be of minimal chromatic number if $\chi(G) = d(G)$. A graph G of density d is d-saturated if the addition of an edge increases $\chi(G)$.

Theorem A d-saturated graph of minimal chromatic number admits a unique d-coloring.

Two points a and b of a graph G are symmetrical if they are both adjacent to the same set of points. A graph G is symmetrical if any pair of its points are symmetrical.

Theorem A graph G of density d is symmetrical if and only if it decomposes into a product of d nonintersecting trivial graphs (i.e. graphs having no edges) and it is determined up to isomorphism by giving the sizes of the graphs.

In present terminology such a graph would be called a complete d-partite graph. The final results of this long paper deal with maximal d-saturated graphs, i.e. graphs that are d-saturated and contain at least as many edges as any other d-saturated graph with the same number of points. Such graphs are characterized. The dimensional polynomials of these graphs are used to bound the dimensional polynomials of other graphs of the same number of points and same density.

Theorem Let G be the maximally d-saturated graph of order n, and let $Q(G) = 1 + a_1 x + \ldots + a_d x^d$ be its dimensional polynomial. Then for any graph G' of density d and order n its dimensional polynomial

$$Q(G') = 1 + b_1 x + \ldots + b_d x^d \text{ satisfies } b_i \leq a_i, i = 1, \ldots, d .$$

In Ref. Z21 Zykov treats questions of composing graphs whose points and edges are labeled by elements that can be combined by an addition operation $\oplus$. The composition of graph A with graph B is a new graph obtained by identifying some points of A with some points of B subject to

(1) No two distinct points of A are to be identified with the same point of B.

(2) Whenever the endpoints of two edges are identified, the edges themselves must be identified.

(3) The labels on the points and edges that are identified are subject to the $\oplus$ operation in the same order in which the identifications are made.

Note that this composition operation is very similar to the addition operation of the previously discussed paper. By taking linear forms $a = r_1 A_1 + \ldots + r_m A_m$ where the r_i are elements of a unitary ring and the A_i are graphs, a module is formed. This module is then equipped with a peculiar type of multiplication which makes it into an algebra. The multiplication is defined as follows.

The product of two basis elements A and B is the linear form of all possible compositions of the first graph with the second one. Examples are given in the paper. The product is extended to arbitrary linear forms by requiring the distributive law to hold. This algebra is called the basic algebra of graphs. Quotient algebras are constructed by using the ideal generated by graphs of no more than n-1 points. The quotient algebra is called the algebra of complexes modulo n.

Theorem The set of all connected graphs having less than n points forms a system of generators for the algebra of graphs modulo n, $1 < n \leq \infty$.

Theorem In a commutative algebra of graphs modulo n $(1 < n < \infty)$ every finite collection of monomials that are distinct elements of the algebra of order $< n$ is linearly independent.

Corollary In a commutative algebra of graphs modulo ∞ the system of connected graphs forms a minimal system of generators, i.e. no connected graph can be expressed by other connected graphs by means of the operation of addition and multiplication of the elements of the algebra or by multiplication by elements of the ring.

The last paragraphs of this paper deal with applications of the algebra of graphs to the study of symmetric functions. This is done by constructing an isomorphism between the algebra of trivial graphs (modulo q + 1) and the algebra of symmetric functions in q variables. (Trivial graphs are graphs with no edges.) These results are extended to what the author calls 2-dimensional symmetric functions, relating them by an isomorphism to the algebra of graphs (modulo q + 1).

Vizing[V22] considers the operation of combining graphs under the Cartesian product. If $G_1 = (X_1, E_1)$ and $G_2 = (X_2, E_2)$ then the Cartesian product $G_1 \times G_2$ was defined in Sec. III-A in the discussion of Vizing and Plesnevitch's paper.[V27] Theorems of Vizing's paper relate certain properties of the product graph to the same properties of the graphs making up the product.

Theorem Let $\chi(G), \chi(H)$ be the chromatic numbers of G,H. Then $\chi(G \times H) = \max\{\chi(G), \chi(H)\}$.

Similar theorems give bounds for the coefficients of internal and external stability of $G \times H$ in terms of these parameters of G and H. Other theorems relate the lengths of paths in the product graph to the

length of paths in the factors. A Hamilton path goes through all points of the graph. A Hamilton cycle is a closed Hamilton path.

Theorem If G has a Hamilton cycle and H has a Hamilton path, then $G \times H$ has a Hamilton cycle.

A graph is prime if every factorization of it contains at most one factor that is not a one-point graph.

Theorem Let G, H, F, K be graphs. If $G \times H \sim F \times K$ and $H \sim K$, then $G \sim F$. (Here $\sim$ stands for isomorphism.)

Two factorizations are equivalent if the factors are isomorphic in some order.

Theorem Any two prime factorizations of a connected graph are equivalent.

This paper by Vizing is closely related to some work of Sabidussi.[37]

Zaretskii,[Z9] in a follow-up to the preceding paper, exhibits a graph with six components having two distinct factorizations into prime graphs to show that Vizing's last mentioned theorem does not extend to nonconnected graphs. He also points out that six is the least number of components such a counterexample could have.

Zaretskii[Z10] has also considered problems related to the relation between Hamilton lines and circuits in component graphs and the existence of such configurations in their product. Necessary and sufficient conditions are given that a graph G must satisfy in order that there exist a graph H in some previously given class of graphs such that $G \times H$ has a Hamilton line or circuit. Typical of these results if the following

Theorem The following are equivalent:

(1) G is connected and has at least two points

(2) H exists that has no Hamilton ircuit and such that $G \times H$ does have a Hamilton circuit

(3) H exists having no Hamilton line and such that $G \times H$ does have a Hamilton line

(4) H exists having no Hamilton line and such that $G \times H$ does have a Hamilton circuit.

Other theorems of the paper use the concept of an n-cover. A star S_n is a tree with n edges, n points of degree 1 and 1 point d degree n. An n-cover of a graph G is a partial graph of G, each component of which is an m-star with $m \leq n$. The number d_G^* for a graph G is defined as follows. If M is a set of points in G, let $f_G(M)$ be the points of G adjacent to at least ne point of M. If M is internally stable in G, define the degree of M to be $d_G(M) = \frac{|f_G(M)|}{|M|}$. Let $d_G^* = \min d_G(M)$, where the minimum is taken over all internally stable sets M of G. d_G^* is called the lowest degree of G.

Theorem G has an n-cover if and only if $d_G^* \geq \frac{1}{n}$ (for $n \geq 2$).

Theorem Let G be connected, $|V(G)| \geq 2$, $x_0 \in V(G)$ and $G-\{x_0\}$ has a 2-covering. Then there exists an n such that $G \times P_{2n+1}$ has a Hamilton line. Here P_k is a path of length k and V(G) is the point set of G.

Theorem The following are equivalent

(1) G is connected and either $|V(G)| = 1$ or at least one of G, $G - \{x\}$ has a 2-covering.

(2) G is connected and either $|V(G)| = 1$ or one of G, $G-\{x\}$ has a 2-covering.

(3) There exists H with at least 2 points of degree 1 such that $G \times H$ has a Hamilton line.

(4) There exists a tree H such th t $G \times H$ has a Hamilton line.

Many other similar theorems are proven.

Lvovskii[L20] has considered conditions under which the state graph of an automaton can be regarded as a subgraph of the Cartesian product of the state graphs of what he calls "elementary" automata.

Melikov et al.,[M18, M19, M20, M21, M22] have also considered relations between problems in automata theory and graphical operations on their state graphs. The operations of union, intersection, direct product, and Cartesian product are defined.

Theorem The direct product of two graphs equals their Cartesian product if and only if they are saturated. (By a saturated graph the author means the graph corresponding to the universal relation on a set.)

The final operation defined is a complicated sum operation. It is shown that a saturated graph of n points is a sum of n one-point, one-arc graphs.

The question in automata theory of whether a given automaton can be expressed as a product of parallel automata is closely related to that of determining whether a given graph has a nontrival factorization as a direct product. A fairly simple criterion is given for a graph to have such a decomposition. The method attempts to put the adjacency matrix of the graph into a certain form that is called a "regular block matrix." An example is given, but the author points out that his criteria are difficult to apply if the graph is very large or if the graph is regular. The results are extended to multigraphs with the same limitations obtaining.

In his latest paper Melikhov extends his results on the decomposition of a direct product to the decomposition of a Cartesian product. The criterion for factorization that is derived is similar to the one for factorization into a direct product. Again an algorithm is given for finding a Cartesian factorization. The efficiency of the algorithm is rather limited.

Zykov has studied a class of graphical questions for which no directly comparable Western work exists. In Ref. Z27, where the only discussion in English of this work appears, Zykov refers to it as the study of "recursively calculable functions of graphs." This work involves performing certain operations on graphs that recursively decompose them. Certain properties of the original graph are then given recursively by properties of the graphs appearing in the decomposition. The final result of the decomposition yields certain graphs, such as trivial graphs, for which the properties can be easily determined. Let us first consider a simple example and then turn to a more detailed study f this method.

Let $D_G(t)$ be the dimensional polynomial of a graph as discussed earlier in this section. Let a be some point of G, and let $L_p(G)$ be the subgraph of G obtained by removing the point p and all points not adjacent to it. Let $L_r(G)$ be the subgraph remaining after removing the point a. Then the following recursion formula holds.

$$D_G(t) = t\, D_{L_p(G)}(t) + D_{L_r(G)}(t) \quad .$$

By performing the operations on $L_p(G)$ and $L_r(G)$ recursively, an expression sion is finally derived for $D_G(t)$. Thus certain characteristics of a graph G, the numbers of complete subgraphs of each order, is recursively calculated through the decomposition procedure described above. We turn now to a more general discussion of Zykov's method.

In Ref. Z20 Zykov considers the following operations on a graph G. Let a be a point of G, let $L_p(G)$ and $L_r(G)$ be as defined above, and let $L_q(G)$ be the graph generated by points different from a. Let K be a ring with unit e and H an additive group with K as operators. Let Φ map graphs into H and satisfy

$$\Phi(G) = p\Phi(L_p(G)) + q\Phi(L_q(G)) + r\Phi(L_r(G)) + S$$

with $\Phi(E_n) = \Phi_0$, where E_n is the trivial graph on n points and $p,q,r \epsilon K$, $S \epsilon H$. By requiring that $\Phi(G)$ be independent of the labelling of the points, Zykov shows that $\Phi(G)$ depends only on the number d_i of complete i-point subgraphs of G and on $\bar{d}_i$, the number of internally stable sets of points of cardinality i.

In Ref. Z22 Zykov introduces operations on the edges of a graph G. Let L_α delete some chosen edge (a,b) of G, L_{δ_1} and L_{δ_2} delete the edges incident to points a and b, respectively, and L_λ delete edges incident upon a or b. Let

$$\Phi(G) = \alpha\Phi(L_\alpha) + \delta[\Phi(L_{\delta_1}) + \Phi(L_{\delta_2})] + \lambda\Phi(L_\lambda) + T \quad ,$$

$\Phi(E_n) = 0$. $\Phi(G)$ is called an <u>edge function</u> if its value is independent of the labelling of the edges. The author then shows that if Φ is an edge function then its value depends only on the numbers $\nu_{ij}(G)$ of partial graphs of G having i nonisolated points and j edges.

In Ref. Z23 Zykov extends this technique to operations that involve removing both points and edges. The new operations here are L_β and L_γ, which are obtained from L_α by identifying points a and b such that in L_β the new point is adjacent to all points of L_α that were adjacent to a <u>or</u> b, and in L_γ the new point is adjacent to those points of L_α adjacent to <u>both</u> a and b. The equation for $\Phi(G)$ in this case is

$$\Phi(G) = \alpha\Phi(L_\alpha) + \beta\Phi(L_\beta) + \gamma\Phi(L_\gamma) + \delta[\Phi(L_{\delta_1}) + \Phi(L_{\delta_2})] + \lambda\Phi(L_\lambda) + T,$$

$$\Phi(E_n) = 0 \quad .$$

For $\delta = \lambda = 0$, Zykov shows that if $\Phi(G)$ is independent of the labelling then it is determined by the number of edges of th graph G and the quantities

$$\ell_k(G) = \sum_{i \geq 0} (-1)^i \, p_{i+k+1,i}(G) \text{ and } q_k(G) \quad ,$$

where $p_{ji}(G)$ is the number of partial subgraphs of G having j edges, cyclomatic number i, and no isolated points, and $q_k(G)$ is the number of partial subgraphs of rank k + 1 having no isolated points and only complete subgraphs as components.

In Ref. Z24 these results are used to derive expressions for the chromatic polynomial of a graph. (Zykov calls this the "distribution polynomial.") A result of Whitney[50] is rederived th t says that

$$P(G,m) = \sum_M (-1)^{d_2(M)} m^{k(M)} \quad ,$$

where $d_2(M)$ is the number of edges of M, $k(M)$ is the number of components of M, $P(G,m)$ is the number of colorings of G in m or less colors, and the summation is extended over all partial graphs M of G.

Zykov Z26 extends the above results to the case of multigraphs with loop . The particular function considered is

$$\Phi(G) = \alpha\Phi(L_\alpha) + \beta\Phi(L_\beta) + 1, \quad \Phi(E_n) = 0 \quad .$$

Φ is independent of the labelling if and only if K is commutative. The function $\Psi(G) = 1 + (\alpha+\beta-1)\,\Phi\,(G)$ is introduced and it is shown that

$$\psi(G;\alpha,\beta) = \alpha^{d_2(G)} P\left(G;\frac{1}{\alpha}-1,\frac{\beta}{\alpha}\right) \quad ,$$

where $P(G;x,y) = \sum_{i,k\geq 0} p_{i+k,i}(G)x^i y^k$ and $p_{ji}(G)$ is the number of partial multigraphs of G having j edges and cylomatic number i. The function ψ is related to the dichromate χ of Tutte.[46] Zykov shows that

$$\chi(G;x,y) = (x-1)^{d_2(G)-\ell(G)} P\left(G;y-1,\frac{1}{x-1}\right),$$

where here $\ell(G)$ is the cyclomatic number of G.

Most recently Zykov[Z30] has used his technique of performing operations on a graph to derive recursive formulae for the number of partial colorings of a graph and the number of partial "contractions" of a graph. The latter operations consist of identifying adjacent points of a graph.

These results of Zykov have been extended by three other Soviet workers.

Vitaver[V20] has considered the function $\Phi(G) = \alpha\Phi(L_\alpha) + \mu\Phi(L_\mu) + 1$, where L_α is as before, and L_μ deletes an edge (a,b), identifies its endpoints, and joins that new point to the other points of G that were adjacent to <u>exactly one</u> of a and b. Conditions are given for $\Phi(G)$ to be independent of the labelling of points and edges. Under these conditions it is shown that for $\alpha = 1$, the function $\Phi(G)$ depends on the numbers $p_{ji}(G)$ which were defined earlier

Matyushkov[M13] has extended some of Zykov's work to digraphs. The possible number of operators one can define here is, of course, considerably expanded. For a class of functions that involve deletion of a point it is shown that the function value depends only on the number

of points in internally stable sets of the digraph and the number of internally stable sets of points in the complement of the digraph.

Matyushkov[M14] has also studied the interesting question as to whether information is "lost" when one breaks into two parts the set of operators that act on a graph. Let us denote by $\Gamma(G)$ the set of graphs associated with a given graph by one of the recursive procedures of the type that have been discussed, say $\Gamma(G) = \{G_1, \ldots, G_n\}$. Let $\Gamma(G)$ be broken into parts $\Gamma_1(G) = \{G_{i_1}, \ldots, G_{i_k}\}$ and $\Gamma_2(G) = \{G_{j_{k+1}} \ldots G_{j_n}\}$ such that $\Gamma(G) = \Gamma_1(G) \cup \Gamma_2(G)$. Let $\Phi_1(G)$ and $\Phi_2(G)$ be the functions corresponding to the operations $\Gamma_1(G)$ and $\Gamma_2(G)$. Matyushkov gives three examples, one where the functions $\Phi_1(G)$ and $\Phi_2(G)$ convey less information about G than the original $\Gamma(G)$, one where the information is exactly the same, and one where extra information about G is obtained through the splitting. The examples raise the natural question of finding necessary and sufficient conditions on the ring K, the operator Γ, and the class of graphs in question for which Γ can be "split" so that no information loss occurs.

The most recent paper on recursively calculable functions of graphs is by Narzullaev.[N1] The operator considered here is

$$\Phi(G) = \alpha\Phi(L_\alpha) + \gamma\Phi(L_\gamma) + \lambda\Phi(L_\lambda) ,$$

where L_α and L_γ are as previously defined and L_λ denotes the result of removing from the graph <u>both</u> points a and b together with their incident edges. In this case it is shown that the value of the function Φ depends on the numbers $\bar{r}_{ji}(G)$ which denote the number of colorings of the vertices of G such that

(1) Nonadjacent points have different colors

(2) i colors are used

(3) Exactly j points are colored.

Titov[T2] has considered methods for constructing inseparable graphs and networks. A graph G is inseparable if every subgraph of G has at least two points in common with its complement in G. A system of operations is introduced by which every inseparable graph can be generated from the graph with two points and three parallel edges.

The last paper to be discussed dealing with operations on graphs is by Epifanov.[E2] He considers operations on multigraphs of the following four types.

(1) Replace two edges joining the same pair of points by one edge

(2) Replace a chain of two edges joining a part of points by a single edge joining those points

(3) Replace a triangle with points a, b, c, by a star with edges [a,d], [b,d], [c,d]

(4) The inverse of (3).

The following theorem then settles a conjecture of Akers[2] and Lehman.[27]

Theorem Let G be a planar multigraph with two terminals. In order for there to exist a finite sequence of transformations (1)-(4) that preserves terminals and reduces G to a single edge joining these terminals, it is sufficient that G is finite, and has no isolated points, and that through each of its edges there passes an elementary chain.

I. Properties of Matrices Associated with a Graph

Although several papers discussed in other sections deal at least in part with matrices associated with a graph, this section is separated on the principle that in these papers the whole focus of the paper is on properties of these matrices.

Likhtenbaum[L8] introduces the (0,1) matrix of points and edges C. The entry $c_{ij} = 1$ if and only if point i is incident with edge j. Then he sets $A = CC^T$ and $B = C^TC$. Thus A is related to the usual adjacency matrix but has diagonal entries equal to the degrees of the points. B is related to the usual incidence matrix of edges but has all diagonal entries equal to 2. Traces of powers of these matrices are considered and relationships are given between these traces and the number of what he calls "closed homomorphisms of a polygonal line L_m into the graph." These correspond roughly to closed sequences of points and edges in the graph where points and edges may be retraced several times. The main result of the paper states that traces of equal powers of A and B are equal.

In[L9] these results are extended by giving a more detailed study of closed paths and "antipaths" in a graph. The characteristic polynomials of A and B are introduced and their corresponding coefficients are shown to be equal.

In Refs. L10 and L11 more explicit formulas are worked out for enumerating closed paths in the graphs. In the last section of Ref. L10 Likhtenbaum derives bounds for the maximum density of the union of two graphs where the point sets of the two graphs are not necessarily disjoint. His inequality for the maximum possible density is

$$d \le \sum_{k=2}^{m} \binom{m+n-k-2}{n-2}(k+1) + \sum_{\ell=2}^{n} \binom{m+n-\ell-2}{m-2}(\ell+1) - 1$$

$(m,n = 1,2,\ldots)$. Here m and n are the densities of the two graphs making up the union.

Vakhovskii[V1] has continued the work of Likhtenbaum in studying the matrices A and B. He proves the following theorems.

Theorem The eigenvalues of A and B are real and nonnegative.

Theorem The eigenvalues of A and B do not exceed $\max_{i \neq j}(a_{ii} + a_{jj})$

Theorem The following inequality holds

$$\lambda_q \leq 2 + \sqrt{\max(\alpha_i \alpha_j)} \leq 2 + \max \alpha_i \quad .$$

Here λ_q denotes an eigenvalue of A and/or B and $\alpha_i = \sum_{\substack{k=1 \\ k \neq i}}^{\beta} b_{ik}$.

Theorem If G has α points, then the rank of A is at least $\alpha - 1$.

Numerous Western writers[37,9,24] have also dealt with properties of eigenvalues of the adjacency matrix of a graph.

Khomenko and Gavrilyuk[K23] have also extended the work of Likhtenbaum. They give methods for finding maximal complete subgr·phs of a graph, Hamilton cycles of a digraph, and Hamilton circuits of a graph. The procedures are very inefficient computationally.

Breido[B18] has characterized the state graph of a linear autonomous network in terms of properties of the adjacency matrix. The result is well-known in Western literature.[14,18]

Epshtein[E3] has considered properties of the adjacency matrix of acyclic digraphs associated with the flow of information in a controlled system. Powers of the adjacency matrix provide information on the number of paths of each length between any pair of points.

IV APPLICATIONS OF GRAPH THEORY

This chapter contains a discussion of Soviet papers that are mainly concerned with applications of graph theory. The dividing line between the "pure" and "applied" areas is not sharp for some papers, and thus the classification is somewhat arbitrary in some instances.

A. Problems of Games on a Graph

A kernel of a graph is a set of points that is both internally and externally stable. Games are defined on graphs by associating points of the graph with positions in the game. Arcs of the graph correspond to admissible moves, the players moving alternately. In studying problems of games on a graph, the existence of kernels plays a fundamental role. Varvak[V5] has given conditions under which there exists a kernel in the direct product of several graphs.

Theorem If the finite graphs G_i have kernels, then their direct product $G_1 \times \ldots \times G_n$ has a kernel.

Varvak has given a procedure for finding a kernel in the direct product in terms of the kernels of the factor graphs. Let $\{i_1, \ldots i_k\} \subset \{1,2,\ldots,n\}$. Let $\hat{S}_{i_1 \ldots i_k}$ be the set of $x = (x_1, \ldots, x_n)$ such that $x_{i_j} \in S_{i_j}$ and $\check{S}_{i_1 \ldots i_k}$ the subset of $\hat{S}_{i_1 \ldots i_k}$ satisfying $x_\ell \in X_\ell - S_\ell$, $\ell \neq j$.

Theorem Let graphs $G_1, \ldots, G_n$ have kernels $S_1, \ldots, S_n$ and $x_k \in X_k$ implies $\Gamma_k x_k \neq \emptyset$. Then (1) for $n = 2s+1$ the set $S = \bigcup \hat{S}_{i_1 \ldots i_{s+1}}$ is a kernel of $G_1 \times \ldots \times G_n$ where the union is taken over all combinations of $s + 1$ of n indices and, (2) if $n = 2s$ the set $S = S_1^* \cup S_2^*$ where $S_1^* = \bigcup \hat{S}_{i_1 \ldots i_{s+1}}$ and $S_2^* = \bigcup \check{S}_{1, i_1 \ldots i_{s-1}}$ is a kernel of the product.

The latter union is taken over all combinations of s-1 indices of $\{2,3,\ldots,n\}$.

Varvak[VG] also extends his results to kernels in a Cartesian product of several graphs. (He uses Berge's terminology and speaks of the "sum.") In this paper he uses the concept of a Grundy function of a graph. A Grundy function of a digraph $G = (X,\Gamma)$ is an integer-valued function $g(x)$ such that $g(x) \geq 0$ and $g(x)$ is the smallest integer ≥ 0 that is not in the set $g(\Gamma x) = \{g(y) : y \epsilon \Gamma x\}$. Varvak derives an expression for a Grundy function of a Cartesian product in terms of the Grundy functions of the factor graphs. Since the points on which a Grundy function takes the value 0 from a kernel of the graph, Varvak is thus able to find a kernel in the Cartesian product. He also characterizes the win, lose, and draw positions in a Cartesian product.

Butsan and Varvak[B25] have also studied the Grundy function of a direct product of graphs. They show that the Grundy function of a Cartesian product is the minimum of the Grundy functions of the factors. They also show constructively that a Cartesian product has a kernel if at least one of the factors has a kernel. The result holds even for locally finite digraphs.

Brudno[B19] has applied graphs in a trivial fashion to the study of games with complete information.

B. Extremal Path Problems

Problems of finding longest or shortest paths in edge-weighted graphs arise in several areas of application, including scheduling according to project diagrams (PERT charts), routing in road and communication nets, sequencing of the steps in production processes and computer programs, and determining delays in sequential computer circuits. These same applications frequently generate related questions concerned with

the existence of paths between arbitrary points of a graph (i.e. whether the graph is connected or not), and the existence of circuits that might have been introduced inadvertently in the course of writing down the graph. Because the techniques for solving these problems are essentially the same for all applications, they are discussed together in this section.

A series of Soviet papers discusses the use of PERT diagrams as an aid to management for carrying out complex projects. The points of the graph represent tasks that must be completed, and an arc of the graph from a to b indicates that task a must be completed before task b. Completion times are associated with the arcs. Often maximum, minimum, and actual expected completion times are introduced.

Petrova and Karanaukhova[P5] give an algorithm for finding critical paths in a PERT diagram. These paths represent sequences of tasks such that delay in completing any of them causes delay in the earliest time at which the project could be completed. In graphical terms this amounts to finding longest paths in the network. The result of the algorithm is to give for each point k the length T_k of the longest path starting at k, and the length T_k' of the longest path ending at k. From these numbers the ranges of permissible finish times for each task to avoid delaying the project can easily be calculated. Computer realizations of the algorithm are discussed.

Sigal and Chebakov[S16] have worked out an algorithm for solving two problems in the analysis of PERT diagrams: the usual problem of finding a critical path, and the problem of determining the time reserve (or time-excess) associated with each point of the graph, where each arc is weighted with a delay time. Dynamic programming techniques are used.

Radchik[R1] has also treated the optimization problems that arise in the solution of PERT diagrams having two parameters: the time and the cost of carrying out each operation. Both of these optimization problems

are convex programming problems. To minimize the total cost subject to a maximum elapsed time, he offers a solution based on the method of steepest descent. To minimize the elapsed time for a given cost limit, a special method of step-by-step improvement is proposed.

For a general discussion of optimization problems on PERT diagrams, including stochastic nets and the modification of diagrams to satisfy various criteria, see Altaev, et al.[A6]

Bagrinovich and Rabinovich[B1] discuss similar questions. They also discuss the problem of detecting closed paths that have inadvertently been introduced into large-scale PERT diagrams.

Avdeev and Nikolaeva[A9] and Pospelov and Teiman[P10] have discussed PERT diagrams as an aid to project control. Some discussion of "slack time" is included. Also, problems of overall project cost minimization as a function of required completion time are discussed as a linear programming problem.

Zadykhailo[Z1] has also described an efficient procedure for finding critical paths in a graph. Using the implementation he describes, a number of machine cycles that is proportional to the number of edges of the graph is required. He mentions that the procedure can be modified to find shortest paths also.

Maizlin[M1] has described an algorithm for finding which, if any, of a set of words $x_0, x_1, \ldots, x_n$ are equal to those of a given set $y_1, \ldots, y_m$. This same algorithm is used for finding longest delays to each point of a PERT diagram, and also for finding latest allowable times for activities that still permit completion in a minimum time for the overall operation.

Butrimenko[B23] and Butrimenko and Lazarev[B24] have considered the problem of finding shortest paths in a graph when an arc is removed. A labelling procedure is given for finding shortest paths from a prescribed point. A technique is then described for making local changes

in this labelling when some arc is removed. There is a discussion of how this technique can be applied to problems in communication systems. The routing control is decentralized, since only local decisions have to be made.

Burkov[B22] has treated the following graphical representation of certain job scheduling problems. Let the points of a digraph be partitioned into k classes $\pi_1, \ldots, \pi_k$. On one step of the dec .position only sources of the digraph can be removed. Numbers ℓ_i, $i = 1, \ldots, k$ are given so that on any step at most ℓ_i points of class π_i can be removed. The problem is to remove all points in a minimum number of steps. The problem is solved for a few special classes of graphs where $k = 2$ and all graphs consist of several independent paths, the points of each path being partitioned into three classes π_1, π_2, π_1 in that order.

The same paper treats graphs that the author calls pseudo-potential graphs, having the property that all of their Hamilton circuits have the same length.

<u>Theorem</u> If $G = (X,U)$ is pseudo-potential, then there exist numbers α_i, β_i prescribed for each point such that the length of the arc (i,j) is given by $\ell_{ij} = \alpha_j - \beta_i$. The numbers are unique up to a constant.

Finally, certain polynomials are associated with graphs that give information on the minimum number of steps required for their decomposition.

Shkurba[S9] has discussed sequencing and scheduling problems from the point of view of what he calls a <u>theory of ordering</u>. An initial finite partially ordered set that is represented by a digraph $G_0 = (X_0, \Gamma_0)$ is associated with the problem. Certain rules are prescribed that allow the addition of arcs to form a new graph $G = (X_0, \Gamma$ in such a way that some functional $F(G)$ is extremized. One seeks expansions G of G_0 on which this extreme value is taken on.

Shkurba points out how sequencing problems can be cast in th·· form. The assignment problem and the travelling salesman problem also fall in this category. This paper also contains a partial bibliography of other Soviet work on problems of this type.

Mudrov[M28,M29] has given a formulation of the travelling salesman problem by the method of integer linear programming. This problem is to find a shortest Hamilton line or circuit in an edge-weighted graph (usually a complete graph). Later work on the travelling salesman problem claims shorter algorithms and the generalization to two or more salesmen.[P2]

Ermolev[E4] has considered several problems that can be posed as problems of finding optimal paths in a suitably defined graph subject to constraints, for example (1) the travelling salesman problem, (2) standard shortest-path problems, and (3) assembly-line balancing problems. His general statement of the problem is as follows. Let (X,E) be a graph, and to $i \in X$ let there correspond a set Φ_i and a function $f_i(\mu_i)$ defined on paths ending at i. A path $\mu_{i_m} = [i_0, i_1, \ldots, i_m]$ is admissible for i_m if $f_{i_k}(\mu_{i_k}) \in \Phi_{i_k}, k = 1,2,\ldots,m$. The problem is to find an admissible path from a set $A \subset X$ to a set $B \subset X$ for which $f_{i_m}(\mu_{i_m}) \in \Phi^*_{i_m}$, $\Phi^*_{i_m} \subseteq \Phi_{i_m}$, having shortest length $\ell(\mu)$. Two labelling algorithms are discussed for solving this problem.

Kozyrev[K33] considers networks that are acyclic digraphs $G = (X,\Gamma)$. He is concerned with the evaluation of functions f on X such that $f(y)$ is calculable if $f(x)$ is known for $x \in \Gamma^{-1}y$. The points are weighted by a function $g(x)$ and arcs are weighted by $\ell(x,y)$. The function f is prescribed at the sources of the digraph, and then must be extended to the rest of the points. For very large graphs the practical problem of implementing this on a computer is that of keeping all the information in fast-access memory. Kozyrev discusses a technique for (1) partitioning the graph into smaller graphs, (2) solving the problem for these

pieces, and then (3) patching these partial solutions together to solve the whole problem. He mentions applications to problems of finding maximal-weight paths in the graph and problems of dynamic programming.

Tolchan[T6] treats the problem of finding extremal paths in multigraphs with loops. An initial point a_0 is chosen and paths from a_0 to all other accessible points are constructed. A weight, denoted by $W(a_0, \ldots, a_m)$, is associated with each path from a_0 to a_m. Paths such that $W(a_0, \ldots, a_m)$ takes on an extreme value are called extremal paths. The height of a point a_m is the weight of a maximum (or minimum)-weight path from a_0 to a_m. If an extremal path is chosen from a_0 to all other points, the author calls these extremal paths a relief of the graph. Two labelling algorithms are given for constructing a relief of a graph.

Tsoi, Mastyaeva, and Tskhai[T11] have given algorithms for finding longest and shortest paths of a graph or digraph. Their procedure will also give lengths of all cycles in the graph and find paths of prescribed lengths.

Sigal and Chebakov[S15] have given a procedure for finding a certain minimum associated with a 3-dimensional matrix. The method can be applied to find shortest paths between points of a graph, but it does not seem to be very efficient.

Stepanets and Vleduts[S24] and Stepanets[S25] have dealt with problems of finding cycles in a graph having prescribed properties. In Ref. S24 the authors give an algorithm for finding a maximum set of linearly independent cycles. The algorithm labels all points with their distance from some fixed point. Then local operations suffice to determine which points and edges lie in cycles. The algorithm also locates all those edges that do not lie in any cycle.

In Ref. S25 Stepanets considers the problem of finding independent sets of cycles in a graph such that the sums of their lengths are maximal or minimal. He refers to these bases respectively as maximal and minimal bases of vector cycles.

<u>Theorem</u> Let B_1 and B_2 be minimal (maximal) basis systems of vector cycles of a graph G. Then there exists a length-preserving mapping ψ of the set of cycles $B_1, \ldots B_{\nu(G)}$ of B_1 onto the set of cycles $B_1', \ldots, B_{\nu(G)}'$ of B_2. Here $\nu(G)$ is the cyclomatic number of G.

The theorem is proven by finding a perfect matching of a certain bipartite graph that the author assigns to a pair of bases of vector cycles.

Let $G = (X,U)$ denote the graph and $U_z \subseteq U$ denote the set of cyclic edges of G. For an edge $u_j \in U_z$ let $S^-(u_j)$ $S^+(u_j)$ denote some cycle of minimal (maximal) length to which u_j belongs.

<u>Theorem</u> For all systems of vector cycles $S^-(u_1)$, $S^-(u_2)$ ..., $S^-(u_k)$ where $u_1 \in U_z$, $u_2 \in U_z - S^-(u_1)$, ... $u_k \in U_z - \bigcup_{j=1}^{k-1} S^-(u_j)$ there exists a minimal basis system of vector cycles B such that $S^-(u_j) \in B$, $j = 1, 2, \ldots, k$.

<u>Corollary</u> For $k = \nu(G)$ the system of vector cycles chosen as above is a minimal basis of vector cycles in the graph G. (Similar results hold for a maximal basis of vector cycles.)

Tolchan[T5] and Ushakov[U1] have given procedures for testir whether paths exist between any two points of a graph, i.e. for testing whether or not the graph is connected. Tolchan's method involves a labelling procedure starting out from some initial point. Ushakov's method involves the calculation of Boolean powers of the adjacency matrix of the graph, a procedure that is not very useful for large graphs.

Vasilev[V8] has proposed the use of an electrical network for the solution of critical-path problems on a graph. The simulating network

contains voltage and current sources, and diodes are used to represent the directed branches of the graph. Under these conditions, an externally applied current will always flow through the network along a critical path.

Zhdanov et al.[Z12] have also developed algorithms and a computer program for the solution of critical-path problems on large graphs. However, they employ a different model--one in which the arcs of the graph represent the operations to be performed, the points are labelled with the start-stop times, and the arcs are labelled with the durations of the corresponding operations.

Other Soviet work on the detection and enumeration of extremal paths in graphs has been conducted by Lyusternik and Maizlin[L24], Adelson-Velskii and Filler[A2], Lominadze et al.[L13] Epshtein.[E3]

C. Network Flow Problems and the Transportation Problem

Network flow problems are generally concerned with assigning flow values to the arcs of a digraph having two distinguished nodes in such a fashion that (1) flow is conserved at all intermediate nodes and (2) flow values are constrained by certain prescribed arc capacities. The goal is to maximize the flow, or minimize the cost of a prescribed flow when costs are associated with the arcs of the network. This section discusses problems of this type and other related problems.

Tsetlin[T10] treats the following problem. Let numbers $q_1, \ldots, q_n$ be associated with the points of a graph (he calls this a <u>loading</u> of the graph). Let $q = (q_1, \ldots, q_n)$ and $p = (p_1, \ldots, p_n)$ be two loadings of G and assume that $\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i$. Then q can be changed to p by a series of elementary transports of one unit to an adjoining point. A combination of these elementary transports is called a <u>transport schedule</u>, and the number of elementary transports making it up is its <u>cost</u>. He treats the problem of finding minimal-cost schedules for transforming the loading q to the loading p. A simple algorithm is first given for trees and then

for a single cycle. Assuming, without loss of generality, that $\sum_{\alpha=1}^{n} q_\alpha = 0$, a schedule on a cycle of $b_1 \geq b_2 \geq \ldots \geq b_m$ is called <u>regular</u> if $b_{[\frac{n+2}{2}]} \leq 0 \leq b_{[\frac{n+1}{2}]}$. The b_i are values of the transport schedule on the cycle.

<u>Theorem</u> In a graph every edge of which lies in a cycle, a transport schedule is minimal if and only if the schedule for every cycle of the graph is regular.

The procedure for constructing minimal-cost schedules then first prescribes transports on acyclic edges, and then finds regular transports for those edges that are cyclic. The author states as a final result the

<u>Theorem</u> For a given loading q of a graph G there exists a spanning tree D of G such that a minimal cost schedule on G coincides with a minimal-cost schedule on D.

Vizing[V21] considers the relation between flows of equal magnitude on a network. He defines a distance $\rho(\varphi,\varphi')$ between two flows of equal value and proves the

<u>Theorem</u> Let φ be a flow on a network (X,U). Then there exists a flow φ' of equal magnitude with $\rho(\varphi,\varphi') > 0$ if and only if there exists a φ-cycle. (The definition of a φ-cycle was not very clear, but it seems to be a set of arcs of the network that constitutes a cycle of the underlying undirected graph along which flow values can be changed and still remain feasible.)

This theorem is used to prove the purely graph-theoretical result that if two p-graphs (points can be joined by up to p edges) have equal degrees at this point, then one can be transformed into the other by a finite sequence of transpositions indicated in Fig. 3.

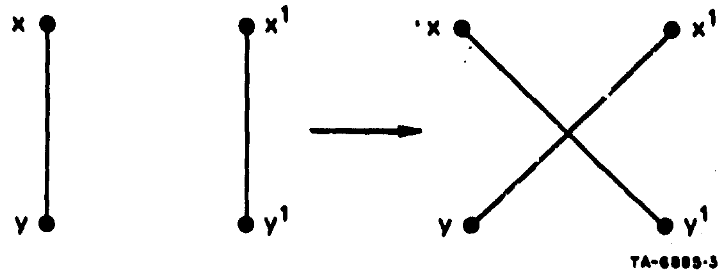


FIG. 3 TRANSPOSITION OF EDGES

He also gives conditions under which one maximum matching of a bipartite graph can be transformed into another by a sequence of transpositions.

Other utilizations of graph theory for the solution of the standard flow problem have been reported by Padalko[P1] for the optimization of electrical power transmission networks; Guseinov et al.[G20] for the scheduling of equipment repairs in electrical power stations, taking into account the load distribution in the power system; Prokofev and Neiman[P13] for the solution of dynamics problems in large automatic, hydraulic systems; Lominadze[L14] for the balancing of assembly lines; and Leonas and Motskus,[L3] who propose a sequential search method for the solution of the nonlinear programming problem that arises when the "cost" function in the optimization is a non-linear function of the branch costs (weights on the arcs).

Burkov[B20] and Lovetskii[B21] have considered network flow problems with the additional constraint of having capacities prescribed on the points. They give an algorithm for finding maximal flows in this case. For planar networks a simplified algorithm is given. Finally, they give some results on the maximum connectivity of a graph in terms of the number of points and edges in it. Ford and Fulkerson[16] have also considered the problem of network flows with constraints on the points.

Khoang[K22] has written a long paper exploring the relation between graphs, network flows, and transportation problems. Vizing's results on transforming graphs of equal degrees into one another and changing maximum

matchings of bipartite graphs into one another are independently derived. The paper concludes with some discussion of Euler lines and Hamilton circuits of graphs.

Gorelik and Shtilman[G14] give a method for solving the network transportation problem that involves distributing goods from producers with supplies A_i to consumers with demands B_j. They claim that their method has certain advantages, one of which is to reduce the amount of matrix manipulation involved.

Evstigneev[E11,E12,E15,E16] has considered transport problems where the goal is to move a certain amount of material through the network in minimum time. Capacities and lengths are prescribed for all arcs. The time required to pass a unit load through an arc is proportional to its length. He considers the minimum time needed to transport an amount P across the network subject to deleting certain arcs of the network. The <u>specific weight</u> $\sigma(P,v)$ of an arc v with respect to the load P is the difference $\sigma(P,v) = T' - T \geq 0$, where T is the minimum time with arc v present and T' is the minimum time with arc v deleted. The <u>relative specific weight</u> of the arc v is $\lim_{P \to \infty} \frac{\sigma(P,v)}{P} = \sigma(v)$. He determines $\sigma(P,v)$ and $\sigma(v)$ as functions of $\varphi(v)$, where φ is the flow value obtained by using the concept of a "parallel transport network."

<u>Theorem</u> The removal of the arc of great relative specific weight $\sigma(v)$ from the network results in the greatest increase in transportation time.

Evstigneev[E14] also considered the problem of "counter-transport" in which each arc of the given network carries not one but two weights that indicate the required nonsimultaneous flow along this arc in each of the two directions. The corresponding graph problem is one of decomposing the graph into two nonintersecting subgraphs of the usual type, such that the transport time of the component graphs will be minimal.

Nikitina and Romanovskii[N5] applied Minty's shortest-path algorithm to the solution of a transport problem that arose in locating cheese processing plants.

Romanovskii has established the mathematical equivalence of six different formulations of the transport problem by showing that each problem can be "embedded" in others through a suitable redefinition of the variables, reexpression of the condition of optimality, etc.

Pshenichnyi[P14] and Mukhacheva[M30] have also given an algorithm for the solution of transport problems using a graph theory approach.

D. Communication Nets

A few Soviet papers on graph theory have arisen from problems in the analysis and synthesis of communication networks. The points and the arcs of the graph represent the stations and the communication channels between stations, respectively, in a direct manner. One problem concerns the optimization or the structure of a communication net (graph) in order to maximize the reliability of transmission (Kiryukhin,[K26,K27]) or to achieve a given flow at minimum total edge cost (Tolchan[T3,T4]). Another problem (Kasimov[K8,K9]) concerns the enumeration of the number of networks having certain redundancy properties. The work of Butrimenko[B23] and Butrimenko and Lazarev[B24] concerned with critical paths and mentioned in a previous section also arose from the study of communication networks.

In an early paper, Povarov[P12] applied matrix methods to the calculation of the lengths and number of paths between any two points in a communication network, and to the determination of the connectivity and "compactness" of the network.

Ambartsumyan[A7] applied some graph theory notions to a problem in the recovery of pulse signals from a mixed sequence of pulse signals and

pulse noise. His solution requires an algorithm for searching for certain subgraphs of the given graph.

Also pertinent is the paper by Kozyrev[K33] described in Sec. IV-B, which discusses the calculation of maximal paths and optimal fluid flows on a very large network.

E. Coding Theory

Graphs have also been utilized in the development of error-correcting codes for communications applications. Some of these applications use the graph of the n-dimensional unit cube, the points and edges of the graph corresponding to the vertices and edges of the cube, respectively. The problem of code design is usually one of selecting a maximal subset of points of this graph that enjoy a prescribed minimal separation (distance), measured along edges of the graph (cube). Tylkin[T14] derived some necessary conditions on a given matrix of distances for it to be realizable by a subset of vertices on this graph. Techniques for the minimization of switching functions also make use of this unit-cube graph (see Sec. IV-H below).

Vinnichenko[V18] has employed a graph in counting the total number of words that can be composed of letters from a prescribed set of alphabets of different lengths. In his representation the points of the graph correspond to the given alphabets, and the points and edges are weighted with the cardinalities of the given alphabets and of their pairwise unions, respectively.

In an important series of papers on the information capacity and other properties of codes for the noiseless channel, Markov[M4,M5,M6,M8] and Levenshtein[L4,L5,L6] have used a weighted, directed graph to represent a code, each point corresponding to a code word and each branch corresponding to a certain "overlap" relation between a pair of code words.

Markov is then able to state, completely in terms of the graph, the necessary and sufficient conditions for a given code to have any of several important properties. Typically, these conditions are expressed in terms of the existence of certain cycles through the root of the graph.

There has been no Soviet work reported on the interesting and important family of Huffman graph-theoretic codes.[25]

F. Computer Programming

All Soviet utilizations of graph theory in computer programming employ the graph to represent the flow chart of a program. That is, the points of the graph represent operations to be performed, and the arcs express precedence relations between these operations. In some cases each point carries a weight, which represents the time of execution of the corresponding operation, or the number of memory cells required for its execution.

In the Soviet literature three types of questions have been asked concerning these graphs. First, imagining that one has several processors for executing the operations, it is required to determine the shortest time (or smallest number of successive operations), and the fewest number of processors, that are necessary to perform all of the operations represented by a graph, taking into account all of the precedence relations among the operations and the various operation times (Bekishev[B5,B6,B7], and Kiknadze[K25]). In particular, Bekishev[B5] has sought partitions of the digraph $G = (X,\Gamma)$ so that (1) $X = \bigcup_{\alpha} S_{\alpha}$ (2) $S_{\alpha} \neq \emptyset$ (3) $S_{\alpha} \cap S_{\beta} = \emptyset$ for $\alpha \neq \beta$ and (4) $\Gamma S_{\alpha} \subseteq \bigcup_{\beta < \alpha} S_{\beta}$. Such a partition is called a A-partition.

Theorem A digraph G has an A-partition if and only if G is progressively finite (i.e., G has no arbitrarily long sequences of arcs. For G finite this is equivalent to G being acyclic). Bekishev has given a simple algorithm for finding an A-partition into a minimum number

of blocks. For a few special cases of digraphs he gives bounds on the maximum number of points that ever need appear in any one block of the partition. He also considers the harder problem of imposing a priori an upper bound L on the maximum number of points that can occur in a block of the partition. For this problem he gives a (0,1) integer linear programming solution for finding an A-partition into the smallest number of blocks.

The second question concerns the actual formulation of an original computation problem in graph terms, and its subsequent transformation and reduction to remove "forbidden" paths--i.e. so that all paths through the graph correspond to permissible computations (Martynyuk,[M12] Smirnov,[S19] Blokh and Neverov,[B11] Blokh and Gorelik,[B12] and Stognii[S26]). Some discussion is given to decomposing the graphs into blocks that simplify the analysis of complicated algorithms. Some roughly comparable Western work has been carried out by Karp and Miller.[26]

Third, one must determine for a given flow chart (graph) the minimum total amount of memory required to execute the corresponding program, and find the optimal allocation of this memory to the different operations (Ershov[E6,E8,E9]).

Finally, we recall the paper of Dambit[D1] on the characterization of graphs having a certain type of "strong basis," as discussed above in Sec. III-B.

G. Networks of Computers

A computer system is a network of elementary computers that are interconnected in a manner to achieve a total computing performance that is substantially greater than that of any of the elementary computers. Computer systems are under intensive study at the Institute of Mathematics of the Siberian Section of the Academy of Sciences of the U.S.S.R. A graph may be used to model such a system in the obvious way: each point

of the graph represents an elementary computer, and an edge or an arc connects a pair of points to indicate that a direct intercomputer communication channel has been provided. The main problems in this area are concerned with the synthesis and evaluation of computer-system graphs that have certain prescribed connectivity properties. Thus, these computer networks employ the same model as is used for communication networks, but in this case the connectivity criteria may be somewhat different.

The contributions of Reshetnyak[R3] on locally path-disjoint graphs and Ziman[Z17] on the embeddability of a graph in a rectangular lattice were discussed in Secs. III-C and III-F. Some recent work along this line has been reported by Radunskii and Grigorovich,[R2] by Zakhrov and Stikhov,[Z4] and by Evreinov and Kosarev.[E10]

H. Combinational Switching Circuits

Graphs have been applied in several constructive ways to model different aspects of switching circuits and their behavior. Lupanov,[L18, L19] Krichevskii,[K35] and others have used graphs to model networks of gate-type logical elements. Graphs have also been used to represent relay-contact networks in the obvious way since the time when these networks were first studied. Special properties have been derived for the graphs of the family of so-called nonrepetitious contact networks--those in which no input-variable label is repeated on more than a single edge (contact) in the graph (Kuznetsov,[K40] Trakhtenbrot,[T7] Vaksov[V3]). Vetukhnovskii[V10] and Oifa[O1] have studied planar and nonplanar contact networks, respectively. Lupanov[L16] and Vetukhnovskii[V9] have counted the number of networks having certain properties by enumerating the numbers of their corresponding graphs. Povarov's work on path enumeration in contact networks[P11] has already been mentioned in Sec. III-E.

A graph can also be used in a simple way to represent a commutation switch--a network of crossbar switches such as is used in a telephone exchange; see Kharkevich and Shvalb.[K19]

Some more unusual applications of graph theory to digital circuitry include the work of Zibin[Z16] on the design of multi-stable triggers (flip-flops); Osis[O2] on the minimization of the number of test points in a complex circuit (see Sec. III-A); and Smolyanitskii[S22] on "rail" circuits.

In estimating the efficiency of algorithms for simplifying disjunctive normal forms of switching functions, Zhuravlev[Z14,Z15] posed the problem of finding maximal cycles in the graph Q_n of the n-dimensional cube. The "cycles" under consideration here have the property that no edges of Q_n join nonadjacent points of the cycle. Equivalently, nonadjacent points of the cycle are at least distance two apart in the graph metric. The cycles are called "snakes" or "snake-in-the-box" codes in Western literature.[12,42] Vasilev[V7] proved:

<u>Theorem</u> For $n = 2^m$, $m = 3,4,\ldots$ there exists a cycle in Q_n of length $2_n/n$.

<u>Theorem</u> For arbitrary $n = 3,4,\ldots$ there is a cycle in Q_n of length $(1-e(n))2^{n-1}/n$ where $e(n) \to 0$ as $n \to \infty$.

These results have been sharpened somewhat by Danzer and Klee.[11]

Glagolev[G3] has derived the following upper bound $\ell(n)$ for the length of the longest cycle in Q_n. He has shown that $\ell(n) < 2^{n-1}$ for $n \geq 4$. This result has been independently derived by Abbott[1] and Singleton.[42]

Levin[L7] has treated questions on the maximal length of cycles in Q_n of another type (not necessarily snakes). He discusses transformations of Q_n of two types. The first involves rotations and reflections of Q_n which he calls transformations of type π. The second type preserves certain inclusion properties of disjunctive normal forms of Boolean

functions which he calls type π'. He attempts to construct longest cycles of Q_n that are acted on transitively and invariantly by transformations of type π and π'. His main result is that the maximum length of a cycle that is mapped transitively onto itself by transformations of type π of Q_n is 2n.

I. Automata and Sequential Switching Circuits

Graphs are used for the representation of the behavior of sequential circuits (or automata, as they are more frequently called in the Soviet literature) in a standard manner: the points of the so-called "state transition graph" or "state graph" represent the internal states, and the arcs represent transitions between these states as induced by the applied inputs. The arcs carry input labels, and either the points or the arcs carry output labels.

The main problems concerning sequential circuits that have been investigated in graph terms in the Soviet literature are: network decomposition (graph decomposition[L20,M19,M21,M23]), the equivalence of automata (graph isomorphism[M22]), combining automata together (sum or product of graphs[M18,M20]), memory reduction and minimization (a certain type of point-reduction process for the graph[K38,G17,G16,G18,L21,L23,T8]), state assignment (a point labelling problem[K7]), and the formation of state diagrams corresponding to a desired input-output behavior (a problem in graph synthesis[Y6,Y5,K6,S1,K1]). As is typical in Western literature, most Soviet papers on sequential circuits employ a directed graph as the means for representing the behavior of the circuit, but very few of them make more than a trivial use of graph theory itself.

As exceptions we mention the work of Kartasheva,[K7] who has discussed the state assignment problem for autonomous (input-free) circuits; Melikhov,[M18,M19,M20,M21,M22,M23] whose several papers on composition and

decomposition of the state graphs of automata have already been discussed; and the following work of Kurmit, Grinberg, Lyubich, Breido, and Zakrevskii.

Kurmit[K38] has evolved several algorithms for solving the problem of finding pairs of states of a given sequential circuit that stand in certain relations to one another--equivalent, congruent, compatible, etc.--in terms of operations on the state graph.

Grinberg[G17] and Grinberg and Lyubich[G16] consider the following graphical problem arising in automata theory. A system of graphs $G = \{(S,\Gamma_1),\ldots,(S,\Gamma_k)\}$ is orthogonal if there exist disjoint sets $\underset{\sim}{S}_i \subseteq S$ such that $\Gamma_i S \subseteq S_i$. Let Σ be the power set of S and let $\tilde{\Sigma} \subseteq \Sigma$ be the smallest subset of Σ enosed under each Γ_i. Let D(G) be the number of elements of $\tilde{\Sigma}$. Previous work has shown that if $D_k(n) =$ max D(G), where the maximum is taken over all systems G with fixed k and $n = |S|$, then $\ell n\, D_1(n) \sim \sqrt{n}\, \ell n\, n$ and $D_k(n) = 2^n$ for $k \geq 2$. The author defines $d_k(n) = \max D(G)$ where the maximum is now taken over orthogonal system of graphs with fixed k and n.

Theorem For $k \geq 2$ $\log_2 d_k(n) \sim n/2$, $d_1(n) = S_1(n)$. These results yield (1) the number of states in a minimal automaton representing a given event E and (2) the number of states in a minimal automaton representing an event specified by a regular expression.

Finally, Breido[B18] has characterized the form of the state graph of a linear autonomous network, and Zakrevskii[Z5] has enumerated the state graphs of autonomous networks.

J. Electric Circuits and Linear Systems

Graphs are used to model electric circuits in three ways. In the first, the points and edges of the graph represent the nodes and branches (circuit elements) of the circuit itself. In this case, the main questions concern problems of layout (see Sec. IV-K) and the formation of

certain types of trees on the basis of which one may write the mesh or node equations. For an early paper describing these concepts see Kudryavtsev.[K36] Fundamental work in this area is due to Kirchoff (1847), Maxwell (1892), and others. For a general description, see Seshu and Reed.[39] Note also the recent work of Geraskin,[G1] who has derived a method for finding all of the two-trees of the graph of an electrical network.

The second way in which a graph may be used to model an electrical circuit, and in general any linear system, is through the use of a so-called <u>signal flow graph</u>, as developed by Mason[31] and Coates.[8] In this directed-graph model the points represent variables, and the arcs express dependencies between variables, with the arrow pointing away from the independent variable and toward the dependent variable. These signal flow graphs may be employed as an analytical aid in solving the linear equations of a system. By rendering conspicuous the nonzero entries, the signal flow graph suggests immediately the order in which the elementary equations of the system should be solved. As in Western work, the numerous Soviet papers that employ signal-flow-graph techniques,[A8,B14,B15,B16,Z13,K28,K29,K34,M10,N3,S17,S31] employ graphs freely as a visual aid, but no real graph-theoretic principles are involved.

Shorin[S11] and Khasilev[K20a] applied signal-flow-graph techniques to the analysis of hydraulic systems.

Finally, any matrix that describes a linear system can be represented as a directed graph in which the points correspond to the variables of the system (the rows and columns of the matrix). These points carry weights that are the diagonal elements of the matrix, and the arcs carry weights which are the off-diagonal elements of the matrix. Blazhkevich[B10] has shown how one may evaluate the determinant of this matrix by a certain set of operations over so-called "pre-trees" of its graph.

LC and RLC circuits are studied in an abstract paper Chausovskii,[C1] who uses for synthesis certain ideas from the spectral theory of nonselfconjugate operators.

K. Circuit Layout

Graph theory has also been applied as an aid in the arrangement of components and conductors of an electric circuit on a circuit board.

Ilzinya and Grinberg[G15, I2] were motivated by the problem of coloring the wires on a circuit board in accordance with certain restrictions--a problem originally proposed in a simplified form by Shannon.[40] In the model employed here, the points of the graph represent individual conductors in a circuit, and an edge is inserted between two points if the wire colors corresponding to these two points are supposed to be different. Thus the wire-coloring problem is reduced to a problem of coloring the nodes of a given graph. In addition to a brute-force solution, the authors have developed some improved algorithms based on the detection of all cliques of the graph. Other wire-coloring problems have been treated in graph terms by Kalninsh.[K5]

Wire layout problems have occupied Ryabov,[R5] who has developed algorithms for arranging the wires and connections on a circuit board in order to minimize the number of wire crossings. (The graph representation in this case is the obvious one. Kharchenko[K20] has also treated the routing problem, and has tried to minimize the lengths of the wires, or the number of wire crossings, or both, using a large grid of square cells to represent the range of possible wire locations on the circuit board. Some discussion is given regarding the best order in which to construct the connecting chains. Tyurenkov[T16] has derived two algorithms for the routing problems, with the goal of minimizing internal wire lengths, using a minimum number of crossings.

Finally, Plesenevich[P6] and Dambit[D3] have each devised algorithms for determining whether or not a given graph is planar, a problem that has an obvious application to the layout of a single-layer electrical circuit.

L. Stochastic Processes

Medvedev[M15] has considered graphs associated with Markov systems. The points of the graph represent states of the system, and the edges are weighted by the transition probabilities p_{ij}. The system is ergodic if $\lim_{k \to \infty} p(i,j,k) = p_j$ independent of i, where $p(i,j,k)$ denotes the probability of moving from state i to state j in exactly k steps. A theorem in probability says that $p_i = D_i / \sum_{i=1}^{n} D$ where D_i is the minor of the matrix I-P obtained by deleting the i-th row and column and $P = (p_{ij})$. Medvedev gives a graphical interpretation of the D_i. For a stochastic graph $G = (J,U)$ let $d(J,U) = \prod_{(i,j)\in U} p_{ij}$.

Theorem $D_i = \sum_{V\in U_i} d(J,V)$ where U_i is the set of all spanning arborescenses of G that are rooted at point i. (An arborescence is a rooted directed tree such that all arcs are directed away from the root

The author points out that if the transition graph is sufficiently loosely connected, this method affords a computation procedure for calculating the D_i that may be more efficient than a direct calculation.

Smirnov[S18] has considered multigraphs with loops in which probabilities $p(\alpha_k^i) \geq 0$ are assigned, where $p(\alpha_k^i)$ is the probability of entering point α by the i-th input and exiting by the k-th output. If s is a path in the graph, then a function f defined on paths is called quasiadditive if there exists a path function φ such that for paths s^1, s^2 and their product $s = s^1 s^2$ we have

$$\varphi(s) = \varphi(s^1)\,\varphi(s^2)$$

$$f(s) = f(s^1) + \varphi(s^1)\,f(s^2) \quad .$$

The problem considered is that of computing the mean value $Mf(\bar{s}) = \sum_{\bar{s}} p(\bar{s})f(\bar{s})$, where the summation is extended over all paths $\bar{s}$ from an input to an output of the graph. Transformations are introduced that reduce the graph to a single node with one input and one output. These transformations delete loops, identify points, and reassign probabilities in such a way that the function $Mf(\bar{s})$ is unaltered.

M. Chemistry

A few references were found on the application of graph theory techniques to the characterization and analysis of chemical compounds.[B4,S2,S8,S21,S24,S27,V29,V30,V31] In particular, in a survey of Soviet and Western work on the use of computers for processing chemical information, Vledits and Seifer[V31] mentioned some graph theoretical concepts such as trees and the duals of planar graphs, but little detail is given.

The structural formula of the chemical compound is normally represented by a point-labeled graph in the obvious way. Using this graphical model, the usual problems are those of searching a long list of chemical compounds for those that have some prescribed structural property, or testing a given compound against a list in order to establish isomorphism. In one case, the points and edges of the graph represented only the carbon atoms and bonds, and the search was carried out on the basis of subgraphs (Smolenskii[S21]).

In a completely different vein, Seifer and Shtein[S21] proposed the use of a certain type of labeled graph for the representation of the main features of the multiphasic plots that display some selected chemical property of a two-ingredient mixture, as a function of the percentage composition of the mixture. These plots are converted to graphs through dualization and the insertion of certain additional nodes. This topological representation is also proposed for the machine handling of chemical searches.

N. Linguistics

Graphs are used in linguistics to depict parsing diagrams. That is, the points of the graph represent words, and the arcs represent certain syntatic relations between the words. In graph terms the problems concerning these graphs are: the formation of homomorphic images (i.e. adjacency-preserving maps; Efimova[E1]), isometric embeddability in the unit cube and text isomorphism in general (Firsov[F2] and Shreider[S12]), and the algorithmic simplification of parsing graphs to facilitate machine translation (Balandina[B2]). Examples of many of these topics are discussed by Markus in Ref. M11.

O. Miscellaneous Applications

Finally, we mention without detailed comment several miscellaneous applications of graphs, some of which are novel and interesting, but none of which make extensive use of the theory. Freidzon[F3] introduced graph theory as an aid in the reliability analysis of a navigational control system. Shedivy[S6] showed how graphs can be used in the solution of certain problems in two-place logic. Zaslavskii's[Z11] (m,n) graphs (see Sec. III-C) arose in the study of models of human memory. Tree-graphs that describe the structure of a computer memory having certain

desirable access properties were devised by Adelson-Velskii and Landis,[A1] and Zaichenko[Z3] used graphs in the design of an excavator. Filippov[F1] studied the "canals" of Mars from a graphical viewpoint, concluding that the canals are of "communication type."

Tani[T1] considered three projections onto planes of a 3-dimensional figure and used graphs associated with these projections to give a procedure for reconstructing the 3-dimensional figure.

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13. ABSTRACT

This report describes the results of a comprehensive technical survey of all published Soviet literature in graph theory and its applications--more than 230 technical articles appearing up to June 1968. The purpose of this report is to draw attention to this collection of results, which are not well known in the West, and to summarize the significant contributions. Particular emphasis is placed upon those results that fill gaps or augment the body of knowledge about graph theory as familiar to non-Soviet specialists.

Although Soviet activity in graph theory and its applications lags behind the corresponding Western work in both quality and quantity, the level of activity is increasing rapidly and there are many excellent Soviet contributions to the theory. The best Soviet work has been concerned with bounds on numerical indices associated with graphs, properties of algebraic structures associated with graphs, and operations on graphs. Very little Soviet work has been reported on connectivity properties of graphs, matroid theory, the exact enumeration of graphs having prescribed properties, isomorphism testing, graph coloring, and the use of graphs for modeling in social sciences. A complete bibliography is given at the end of the report.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT

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